Precalculus with Limits
Test Practice
Analytic Trig

Name Solutions
Period______

Find the exact value of each expression.

1. $\sin^{-1}(0)$

0

2. $\sin^{-1} 0$

0

3. $\sin^{-1}\frac{\sqrt{2}}{2}$

$\frac{\pi}{4}$

4. $\arctan\left(\frac{\sqrt{3}}{3}\right)$

$\frac{\pi}{6}$

5. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\frac{5\pi}{6}$

6. $\tan^{-1}(-1)$

$-\frac{\pi}{4}$

7. Does $\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$? Why or why not?

yes $-\frac{\pi}{6}$ is in range of $\sin^{-1}$

8. Does $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$? Why or why not?

no, $-\frac{\pi}{6}$ does not fall in range of $\cos^{-1}$

9. Does $\tan[\arctan(2)] = 2$? Why or why not?

yes, 2 is in domain of arctan
2 is in range of tan

Find the exact value of the following.

10. $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\boxed{-\frac{\pi}{4}}$

11. $\tan\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\tan\left(\frac{5\pi}{6}\right)$

$\frac{1/2}{-\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$

12. $\sin[\tan^{-1}(-1)]$

$\sin\left(-\frac{\pi}{4}\right)$

$-\frac{\sqrt{2}}{2}$

13. $\sin^{-1}\left[\sin\left(-\frac{7\pi}{6}\right)\right]$

$\sin^{-1}(1/2)$

$\boxed{\frac{\pi}{6}}$

14. $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$

$\cos^{-1}\left(\frac{1}{2}\right)$

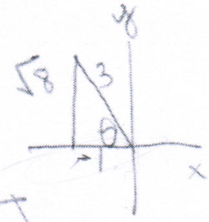
$\boxed{\frac{\pi}{3}}$

15. $\sec(\tan^{-1}\sqrt{3})$

$\sec\left(\frac{\pi}{3}\right)$

$\boxed{2}$

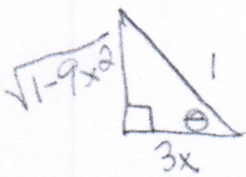
16. $\tan\left(\cos^{-1}\left(-\frac{1}{3}\right)\right)$



$-\frac{\sqrt{8}}{1} = -2\sqrt{2}$

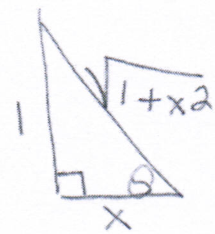
Write the algebraic expressions of the following.

17. $\sin(\arccos\,(3x))$ $= \sin(\theta) = \sqrt{1-9x^2}$



18. $-\csc^2(\cot^{-1}(x))$

$\cot\theta = \frac{adj}{opp}$

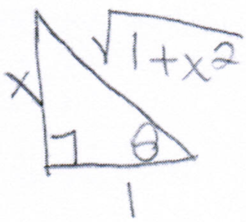


$= -\left(\frac{\sqrt{1+x^2}}{1}\right)^2$

$-(1+x^2)$

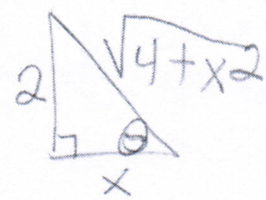
19. $\sin[\tan^{-1}(x)]$

$\frac{x}{\sqrt{1+x^2}}$



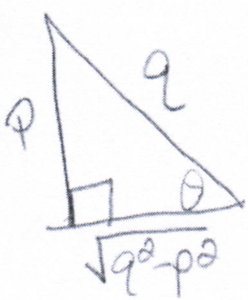
20. $\sin\left(\cot^{-1}\left(\frac{x}{2}\right)\right)$

$\frac{2}{\sqrt{4+x^2}}$



21. $\cot\left[\sin^{-1}\left(\frac{p}{q}\right)\right]$

$\frac{\sqrt{q^2-p^2}}{p}$

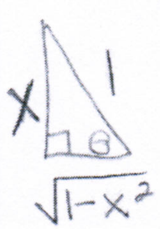


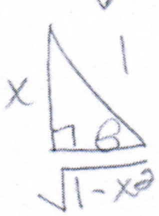
22. $\cos(\sin^{-1}x - \cos^{-1}y)$

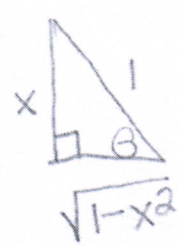
$\cos(\sin^{-1}x)\cos(\cos^{-1}y) + \sin(\sin^{-1}x)\sin(\cos^{-1}y)$

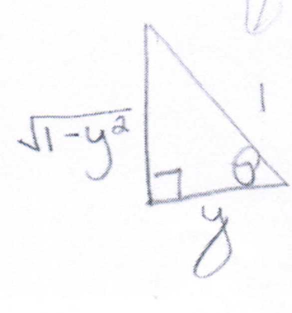
$\sqrt{1-x^2} \cdot y + x \cdot \sqrt{1-y^2}$

23. Show that $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$









24. Given that $f(x) = \cos(x)$, show that $\frac{f(x+h)-f(x)}{h} = \cos x\left(\frac{\cos h - 1}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$.

$$\frac{\cos(x+h)-\cos x}{h} = \frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h} = \frac{\cos x\cos h - \cos x - \sin x \sin}{h}$$

$$= \frac{\cos x(\cos h - 1)}{h} - \sin x \frac{\sin h}{h}$$

Prove the following.

25. $\frac{\tan\theta + \cot\theta}{\sec\theta\csc\theta} = 1$

$$\frac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta\sin\theta}} = 1$$

$$\frac{\sin^2\theta+\cos^2\theta}{\cancel{\cos\theta\sin\theta}}\cdot\frac{\cancel{\cos\theta\sin\theta}}{1} = 1$$

$$1 = 1\ ✓$$

26. $\frac{1+\tan\theta}{1+\cot\theta} = \tan\theta$

$$\frac{1+\tan\theta}{1+\frac{1}{\tan\theta}} = \tan\theta$$

$$\frac{1+\tan\theta}{\frac{\tan\theta+1}{\tan\theta}} = \tan\theta$$

$$\frac{\cancel{1+\tan\theta}}{1}\cdot\frac{\tan\theta}{\cancel{\tan\theta+1}} = \tan\theta$$

27. $\frac{\cos^2\theta - \sin^2\theta}{1-\tan^2\theta} = \cos^2\theta$

$$\frac{\cos^2\theta-\sin^2\theta}{1-\frac{\sin^2\theta}{\cos^2\theta}} =$$

$$\frac{\cos^2\theta-\sin^2\theta}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}} = \cos^2\theta$$

$$\frac{\cancel{\cos^2\theta-\sin^2\theta}}{1}\cdot\frac{\cos^2\theta}{\cancel{\cos^2\theta-\sin^2\theta}} = \cos^2\theta$$

28. $\frac{\sin^3\theta+\cos^3\theta}{1-2\cos^2\theta} = \frac{\sec\theta-\sin\theta}{\tan\theta-1}$

$$\frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)}{1-2\cos^2\theta} = \frac{\frac{1}{\cos\theta}-\sin\theta}{\frac{\sin\theta}{\cos\theta}-1}$$

$$\frac{(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)}{1-2\cos^2\theta} = \frac{\frac{1-\sin\theta\cos\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\cos\theta}}$$

$$\frac{(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)}{\sin^2\theta+\cos^2\theta-2\cos^2\theta}$$

$$\frac{\cancel{(\sin\theta+\cos\theta)}(1-\sin\theta\cos\theta)}{\cancel{(\sin\theta+\cos\theta)}(\sin\theta-\cos\theta)} = \frac{1-\sin\theta\cos\theta}{\sin\theta-\cos\theta}$$

29. Prove the identity.

$$\sec(\alpha+\beta) = \frac{\csc\alpha\csc\beta}{\cot\alpha\cot\beta - 1}$$

$= \dfrac{\frac{1}{\sin\alpha}\frac{1}{\sin\beta}}{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}$

$= \dfrac{\frac{1}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\sin\beta}}$

$= \dfrac{1}{\cancel{\sin\alpha\sin\beta}} \cdot \dfrac{\cancel{\sin\alpha\sin\beta}}{\cos(\alpha+\beta)}$

$\sec(\alpha+\beta) = \dfrac{1}{\cos(\alpha+\beta)}$

$\sec(\alpha+\beta) = \sec(\alpha+\beta)$

Solve the following equations on the interval $[0, 2\pi)$.

30. $\sin(2x) = \frac{1}{2}$

$2x = \frac{\pi}{6}$ $\quad 2x = \frac{5\pi}{6}$

$x = \frac{\pi}{12}$ $\quad x = \frac{5\pi}{12}$

31. $2\sin x + \sqrt{3} = 0$

$\sin x = \frac{-\sqrt{3}}{2}$

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

32. $\cos(2x) + 3 = 5\cos x$ (hint: $\cos(2x) = \cos(x + x)$)

$\cos(x)\cos x - \sin x \sin x + 3 = 5\cos x$

$\cos^2 x - \sin^2 x + 3 = 5\cos x$

$\cos^2 x - (1 - \cos^2 x) + 3 = 5\cos x$

$2\cos^2 x - 5\cos x + 2 = 0$

$2\cos x - 1 = 0$

$x = \pi/3, 5\pi/3$

$\cancel{\cos x - 2 = 0}$

33. $\tan\theta = \cot\theta$

$\tan\theta = \frac{1}{\tan\theta}$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

34. $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad \theta = \frac{\pi}{2}$

35. $\sin^2\theta - 1 = 0$

$\sin\theta = \pm 1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

36. Solve using the graphing calculator.

$19x + 8\cos x = 2$ $\quad x = -.297$ no solution on $[0, 2\pi)$