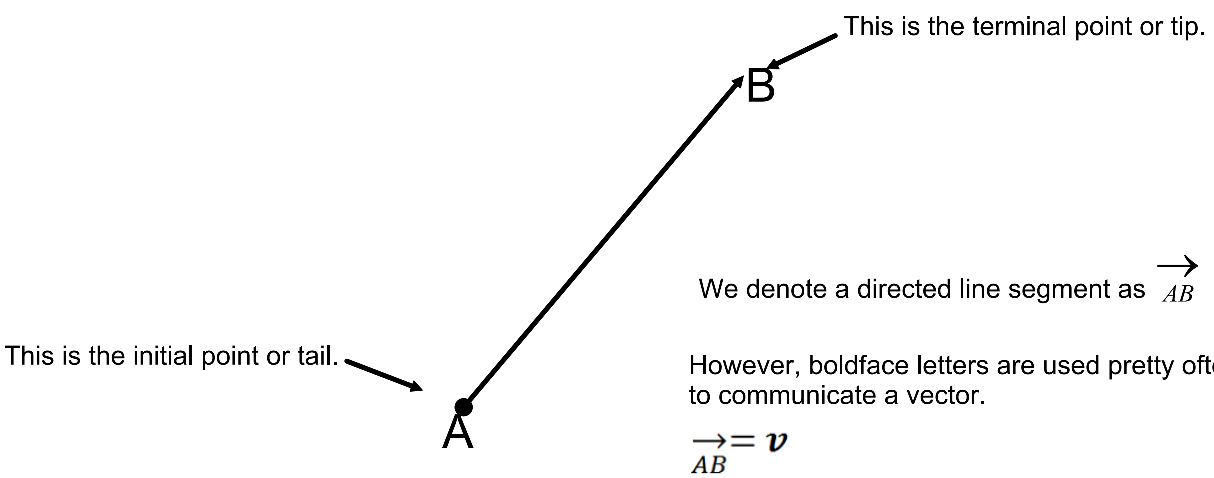
6.3 Vectors #1

In science there are many quantities that are represented by **scalars.** A scalar is just a number/constant representing a quantity. Scalars can describe volume or mass of an object. They can also describe length or area.

There are some quantities that require more information for a complete description. For example, displacement and forces acting on objects need not only a measure of magnitude, but also a direction in which the magnitude is applied. For this we need **vectors.**

Here is a vector. A vector is often described as a directed line segment. Actually, a vector is a *set* of directed line segments all having the same magnitude and direction.



We denote a directed line segment as $\underset{AB}{\rightarrow}$

However, boldface letters are used pretty often to communicate a vector.

$\underset{AB}{\rightarrow} = \boldsymbol{v}$

The **magnitude** of a vector is written as $\left\|\overrightarrow{AB}\right\|$

Example 1
Let **u** be represented by the directed line segment from P=(0,0) to Q=(3,1), and let **v** be represented by the directed line segment from R=(2,2) to S=(5,3).

Show that **u**=**v.**

Component Form of a Vector

The component form of a vector **v** is written as $\langle v_1, v_2 \rangle$ and specifies the horizontal directed distance v_1 and the vertical directed distance v_2 necessary to travel from the initial point to the terminal point.

In order to determine the component form of a vector with initial point (p_1,p_2) and terminal point (q_1,q_2), simply subtract the corresponding "x and y values." This is essentially calculating slope, but writing the change in y and change in x separately.

$$\langle v_1, v_2 \rangle = \langle q_1 - p_1, q_2 - p_2 \rangle$$

Magnitude is calculated using the Pythagorean Theorem (or distance formula for two points).

$$\|v\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}$$

Traditional Distance Formula with two points version of magnitude.

Magnitude written with vector components.

Example 2

Find the component form and magnitude of the vector **v** that has the intial point (-2,3) and terminal point (-7,9).

Example 3

Find the component form and magnitude of the vector **v** that has the intial point (2,-4) and terminal point (5,7).

Note: A **unit vector** has a magnitude of 1 and the **zero vector** has magnitude equal to zero.

Operations on Vectors

The most basic operations on vectors consist of addition and scalar multiplication.

1. $\mathbf{u} + \mathbf{v} =$ $\underline{\quad \mathbf{v} + \mathbf{u} \quad}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} =$ $\underline{\quad \mathbf{u} + (\mathbf{v} + \mathbf{w}) \quad}$
3. $\mathbf{u} + \mathbf{0} =$ $\underline{\quad \mathbf{u} \quad}$
4. $\mathbf{u} + (-\mathbf{u}) =$ $\underline{\quad \mathbf{0} \quad}$
5. $c(d\mathbf{u}) =$ $\underline{\quad (cd)\mathbf{u} \quad}$
6. $(c + d)\mathbf{u} =$ $\underline{\quad c\mathbf{u} + d\mathbf{u} \quad}$
7. $c(\mathbf{u} + \mathbf{v}) =$ $\underline{\quad c\mathbf{u} + c\mathbf{v} \quad}$
8. $1(\mathbf{u}) =$ $\underline{\quad \mathbf{u} \quad}$
9. $0(\mathbf{u}) =$ $\underline{\quad \mathbf{0} \quad}$
10. $\|c\mathbf{v}\| =$ $\underline{\quad |c|\,\|\mathbf{v}\| \quad}$

Vector Addition Definition

If $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$

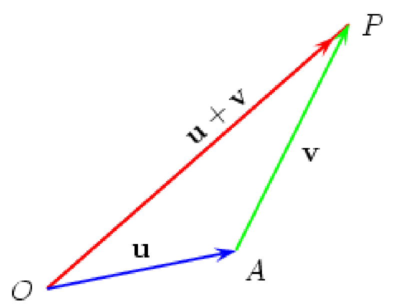
then

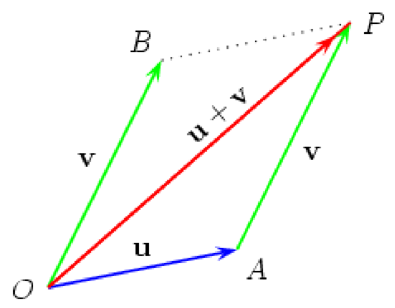
$$u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$$

Scalar Multiplication Definition

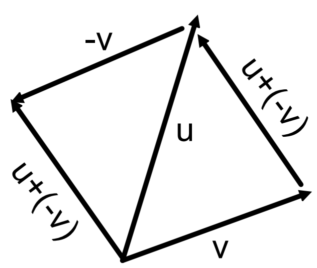
If k is a scalar, then $ku = \langle ku_1, ku_2 \rangle$

Here is a visual of vector addition. One geometric approach is the Triangle Law and the other is called the Parallelogram Law.





Here is a geometric interpretation of **u-v**.



Example

Let $u = \langle 1,2 \rangle$ and $v = \langle 3,1 \rangle$, and find each of the following vectors.

a. **u+v** b. **u-v** c. 2**u**-3**v**

Unit Vectors

A unit vector **u** is a vector with magnitude equal to 1. If you have a nonzero vector **v,** you can find a unit vector **u** that points in the same direction as **v** but has magnitude equal to 1.

Simply multiply the vector **v** by the reciprocal of its magnitude. (This is just scalar multiplication since the magnitude of a vector is just a scalar.)

$$u = \frac{1}{\|v\|} v$$

Example

Find a unit vector **u** in the direction of **v**$= \langle 7,-3 \rangle$

Then verify that the result has magnitude equal to 1.

Consider the following unit vectors

i$= \langle 1,0 \rangle$

j$= \langle 0,1 \rangle$

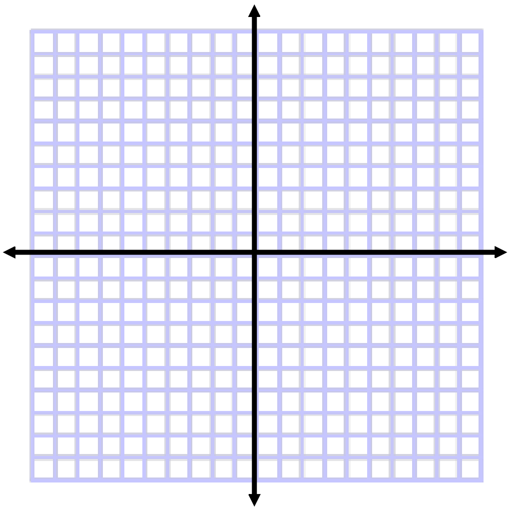
Let's quickly verify that these are indeed unit vectors.

We actually refer to these specific unit vectors as the ***standard unit vector or standard basis vectors.***

We can write any vector **v** as a *linear combination* of two standard basis vectors **i** and **j.**

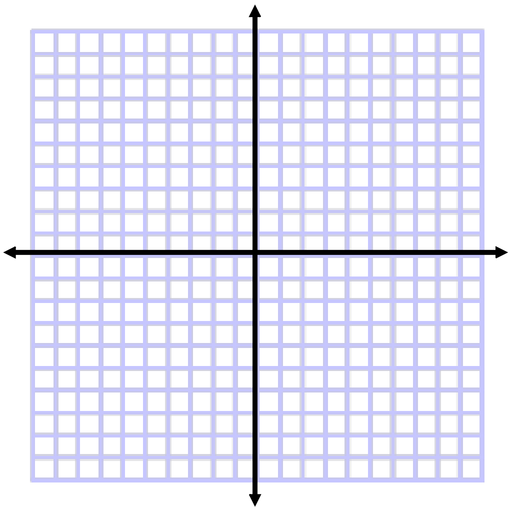
Consider **v**$= \langle 3,5 \rangle$

Draw the graph of this vector.



3**i** +5**j**

Draw the graph of this vector.



So in general we can write any vector **v**$= \langle v_1, v_2 \rangle$ as a linear combination in the following way.

$$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$$

Example

Let **u** be the vector with initial point (-2,6) and terminal point (-8,3).
Write **u** as a linear combinationof the standard unit vectors **i** and **j.**

Example

Let **u** = **i** + **j** and **v** = 5**i** - 3**j**. Find 2**u** - 3**v**