6.3 Vectors #2

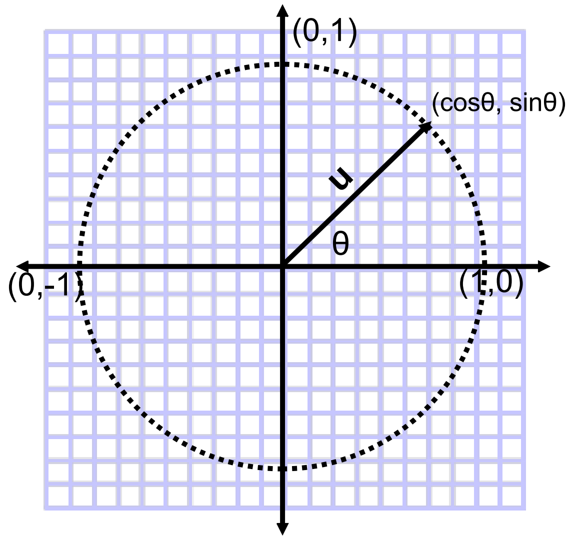
Direction Angles

Let **u** be a unit vector resulting from a counterclockwise rotation on the unit circle.

Then the vector **u** can be expressed as

cosθ**i** + sinθ**j**

We say that angle θ is the ***direction angle*** for the vector.



In general, if we have any vector **v** = a**i** + b**j**, we may write $v = \|v\|\langle \cos\theta, \sin\theta \rangle$

This means that $a\boldsymbol{i} + b\boldsymbol{j} = \|v\|\cos\theta\,\boldsymbol{i} + \|v\|\sin\theta\,\boldsymbol{j}$

$$a = \|v\|\cos\theta$$

$$b = \|v\|\sin\theta$$

We can use this last statement to find the direction angle for our vector! $\longrightarrow$ $\tan\theta = \dfrac{b}{a}$

Example

Find the direction angle for the following three vectors.

a. **v** = -6**i** + 6**j**

b. **v** = -7**i** - 4**j**

c. **v** = 3**i** - 4**j**

Applications of Vectors

Example

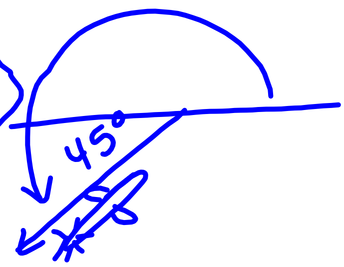
Find the component form of the vector that represents the velocity of an airplane descending at a speed of 100 miles per hour at an angle 45 degrees below the horizontal.

$V = \langle \|v\|\cos\theta, \|v\|\sin\theta \rangle$

$V = \langle 100\cos 225^\circ, 100\sin 225^\circ \rangle$

$V = \langle -100\frac{\sqrt{2}}{2}, -100\frac{\sqrt{2}}{2} \rangle$

$V = \langle -50\sqrt{2}, -50\sqrt{2} \rangle$



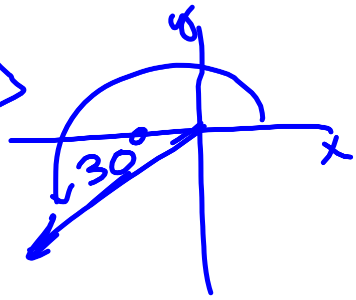
Example

Find the component form of the vector that represents the velocity of an airplane descending at a speed of 100 miles per hour at an angle 30 degrees below the horizontal.

$V = \langle 100\cos 210^\circ, 100\sin 210^\circ \rangle$

$V = \langle -100\frac{\sqrt{3}}{2}, -50 \rangle$

$V = \langle -50\sqrt{3}, -50 \rangle$

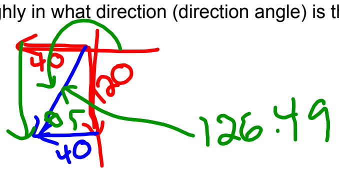


The last two examples are called ***velocity vectors*** and they occur all of the time in physics and calculus.

Example

The pilot of a plane points his airplane due South and flies with an airspeed of 120 m/s. Simulataneously, there is a steady wind blowing due West with a constant speed of 40 m/s.

a) Make a sketch that shows how to find the resultant velocity of the plane.
Roughly in what direction (direction angle) is the resultant velocity?



$\cos\theta = \frac{40}{126.49}$

$\theta \approx 71.56^\circ$

$\tan\theta = \frac{120}{40}$

251.56° = direction angle

b) What is the resultant speed of the airplane?

126.49 m/s

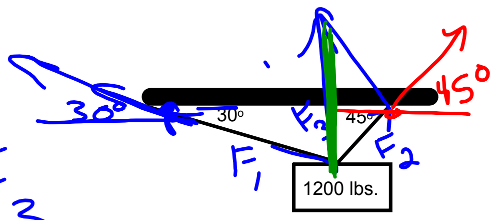
c) After one hour, how far away is the plane from its starting point?

455,364 m

Example

A box of supplies that weighs 1200 pounds is suspended by two cables attached to the ceiling.
What is the tension in the two cables.

There are three forces in this problem
and they are said to be in static equilibrium.



$F_1 + F_2 = F_3$

$F_1 = \|F_1\|\cos 150^\circ\, i + \|F_1\|\sin 150^\circ\, j$

$F_2 = \|F_2\|\cos 45^\circ\, i + \|F_2\|\sin 45^\circ\, j$

$\|F_1\|\cos 150^\circ + \|F_2\|\cos 45^\circ = 0$

$\|F_1\|\sin 150^\circ + \|F_2\|\sin 45^\circ = -1200$

$\|F_1\| = 878.5$ lbs.

$\|F_2\| = 1075.9$ lbs.

$$\|F_1\| = 45 \qquad \|F_2\| = 60$$

$$F_1 + F_2 = F_3$$

$$\|F_3\| = 90$$