Precalculus with Limits
Practice Test
Vectors

Name Solutions
Period________

1. Let vector **v** be defined by the initial point (1,3) and the terminal point (2,5).

a) Write the vector $\boldsymbol{v} = \langle 1, 2 \rangle$

run ↑ ↖ rise

slope $= \frac{5-3}{2-1} = \frac{2}{1}$

b) Determine the direction angle of the vector.

$\tan\theta = \frac{2}{1}$ $\boxed{\theta \approx 63.43^\circ}$

2. Let $\boldsymbol{u} = \langle -1,3 \rangle, \boldsymbol{v} = \langle 2,4 \rangle, and\ \boldsymbol{w} = \langle 2,-5 \rangle$. Calculate the following.

a) 2**u** + 3**v**

$2u = \langle -2, 6 \rangle$
$3v = \langle 6, 12 \rangle$
$+$
$2u + 3v = \langle 4, 18 \rangle$

b) $||\boldsymbol{u} - \boldsymbol{v}||$

$u - v = \langle -3, -1 \rangle$
$||u - v|| = \sqrt{9 + 1}$
$= \sqrt{10}$

c) **v** + **w**

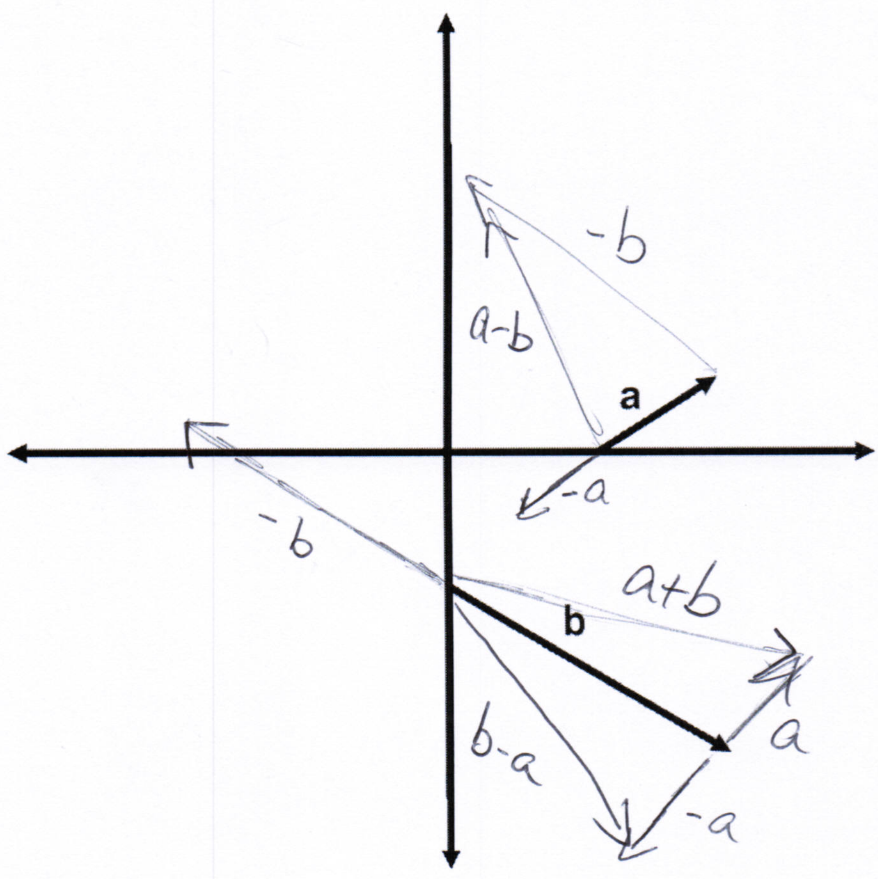
$v + w = \langle 4, -1 \rangle$

3. Use the diagram to sketch the following.

a) **a** + **b**

b) **b** – **a**

c) **a** - **b**



4. Consider the vector $\boldsymbol{v} = -4\boldsymbol{i} + 7\boldsymbol{j}$. Determine a unit vector **u** that is in the same direction as **v**.

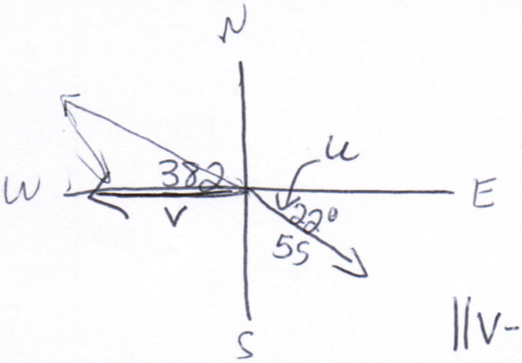
$$\|v\| = \sqrt{16+49} = \sqrt{65}$$

$$u = \frac{1}{\|v\|}v = \frac{-4}{\sqrt{65}}i + \frac{7}{\sqrt{65}}j$$

5. A vector **a** has magnitude $\|\boldsymbol{a}\| = 20$ and has direction angle 210°. Write this vector as a linear combination of the unit vectors **i** and **j**.

$$a = 20\cos 210^\circ i + 20\sin 210^\circ j$$

6. A pilot must actually fly due west at a constant speed of 382 mph. There is a head wind of 55 mph blowing in the direction 22° south of east. What direction and speed must the pilot maintain to keep on course due west?




$v = -382i + 0j$ ← resultant vector

$u = 55\cos 338^\circ i + 55\sin 338^\circ j$

$v - u = -432.995i + 20.603j$

$\|v-u\| = \sqrt{187909.169} \approx \boxed{433.485}$

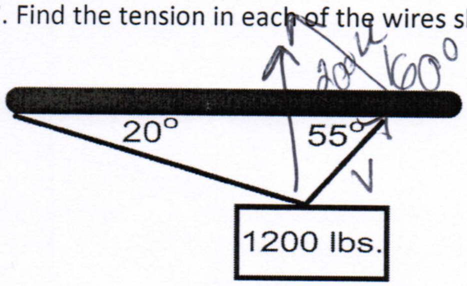
$\tan\theta = \frac{20.603}{-432.995}$

$\theta \approx -2.724$

so add 180°

$\boxed{\theta = 177.276^\circ}$

7. Find the tension in each of the wires shown in the diagram.




$v = \|v\|\cos 55^\circ i + \|v\|\sin 55^\circ j$

$u = \|u\|\cos 160^\circ i + \|u\|\sin 160^\circ j$

$u + v = 0i + 1200j$

$\|v\|\cos 55^\circ + \|u\|\cos 160^\circ = 0$

$\|v\|\sin 55^\circ + \|u\|\sin 160^\circ = 1200$

→ solve system →

$\|v\| = 1167.41$

$\|u\| = 712.57$

8. Find the dot product of the vectors $\boldsymbol{c} = \langle 4, -5\rangle$ and $\boldsymbol{d} = \langle -4, 3\rangle$.

$$c \cdot d = 4(-4) + (-5)(3) = -16 - 15 = -31$$

9. Find the angle between the vectors $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$ and $\boldsymbol{b} = 5\boldsymbol{i} + 12\boldsymbol{j}$.

$$a \cdot b = \|a\|\,\|b\| \cos\theta \longrightarrow \frac{a \cdot b}{\|a\|\,\|b\|} = \cos\theta$$

$$a \cdot b = 3(5) + 4(12) = 15 + 48 = 63 \qquad \frac{63}{65} = \cos\theta$$

$$\|a\| = \sqrt{25} = 5 \quad \|b\| = \sqrt{169} = 13 \qquad \theta \approx 14.25^\circ$$

10. Determine the value of c that will make vectors $\boldsymbol{v} = \langle 5,7\rangle$ *and* $\boldsymbol{w} = \langle -2, c\rangle$ orthogonal.

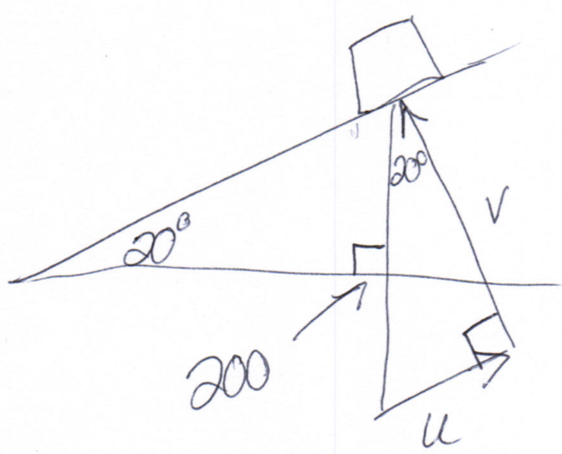
orthogonal means that dot product $= 0$

$$v \cdot w = 0 = 5(-2) + 7(c)$$

$$0 = -10 + 7c$$

$$10 = 7c \qquad \frac{10}{7} = c$$

11. Mr. Carfagna had to pull a washing machine up a 20° ramp on a U Haul truck while moving a friend on Sunday. The washing machine weighed 200 lbs. What force parallel to the incline ramp did Mr. C use to pull the washing machine onto the truck? What was the magnitude of the force perpendicular to the ramp?




$$\sin 20^\circ = \frac{\|u\|}{200}$$

$$200 \sin 20^\circ = \|u\|$$

$$68.4 = \|u\|$$

$$\cos 20^\circ = \frac{\|v\|}{200}$$

$$200 \cos 20^\circ = \|v\|$$

$$187.94 = \|v\|$$