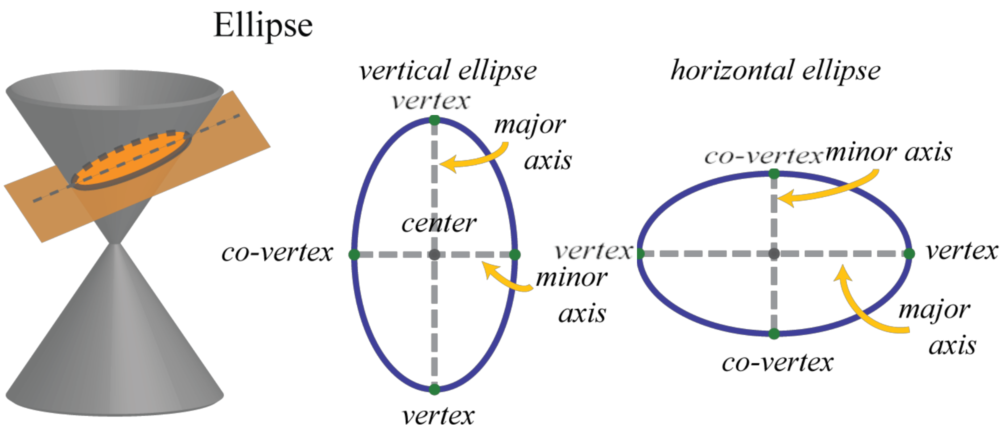
10.3 Ellipses



Ellipse- the set of all points in a plane whose distances from two fixed points (foci) have a constant sum.

The standard equations of ellipses are as follows.

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad \textit{horizontal major axis}$$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1 \qquad \textit{vertical major axis}$$

$0 < b < a$

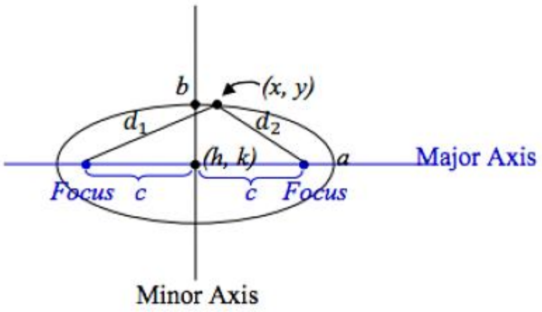
The length of the major axis is 2a.

The length of the minor axis is 2b.

Center is (h, k)

Foci are on major axis and the distance from foci to the center is "*c.*"

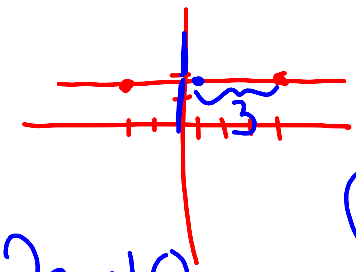
$$c^2 = a^2 - b^2$$



Example

Find the standard form of the equation of the ellipse having foci at (-2,2) and (4,2) and a major axis of length 10.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$(h,k) \to (1,2)$

$$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{16} = 1$$

$2a = 10$

$a = 5$ $c = 3$ $9 = 25 - b^2$ $b = 4$

Example

Sketch the graph of the ellipse given by

$$9x^2 + 4y^2 + 36x - 8y + 4 = 0$$

$9x^2 + 36x + 4y^2 - 8y = -4$

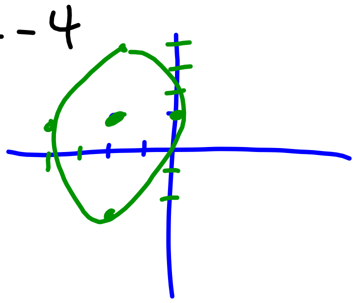
$9(x^2 + 4x) + 4(y^2 - 2y) = -4$

$9(x^2 + 4x + 4 - 4) + 4(y^2 - 2y + 1 - 1) = -4$

$9(x^2 + 4x + 4) - 36 + 4(y^2 - 2y + 1) - 4 = -4$

$9(x+2)^2 + 4(y-1)^2 - 40 = -4$

$$\frac{9(x+2)^2}{36} + \frac{4(y-1)^2}{36} = \frac{36}{36}$$



$$\frac{(x+2)^2}{4 = 2^2} + \frac{(y-1)^2}{9 = 3^2} = 1$$

Example

Sketch the graph of the ellipse given by

$$x^2+4y^2+6x-8y+9=0$$

Example

Find the center, vertices, and foci of the ellipse given by

$$16x^2+25y^2-32x-50y+16=0$$

Example

Find the center, vertices, and foci of the ellipse given by

$$4x^2+y^2-8x-4y-8=0$$

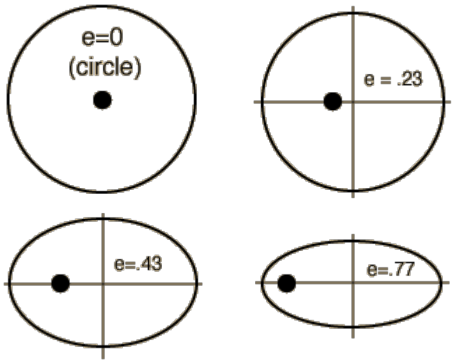
Example

The first artificial satellite to orbit earth was *Sputnik I* (launched by Russia in 1957). Its orbit was elliptical with the center of Earth at one focus. The major and minor axes of the orbit had lengths of 13,906 km and 13,887 km, respectively. Find the apogee and perigee from Earth's center to the satellite. Use these to find the least distance and greatest distance of the satellite from Earth's surface in this orbit. (Earth has a radius of 6378 km.)

Eccentricity

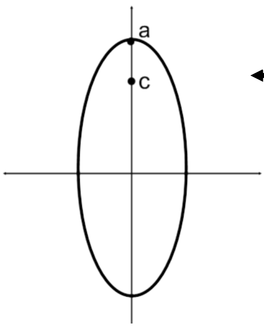
Eccentricity is a measure of how close an ellipse is to being a perfect circle.

The smaller the eccentricity (e), the closer the ellipse resembles a perfect circle. The larger the eccentricity, the more the ellipse will appear stretched along its major axis.

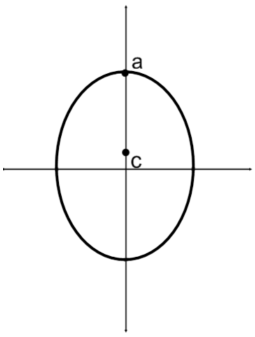


Calculating eccentricity is fairly simple.

It is a comparison of the values of *c* and *a*.



For this ellipse, the value of c is relatively close to the value of a. Therefore, the eccentricity is closer to the number 1. The ellipse is said to be highly eccentric.



For this ellipse, the value of c is relatively far from the value of a. Therefore, the eccentricity is closer to the number 0. The ellipse is said to have a low eccentricity.

Formula for calculating eccenticity.

$$e = \frac{c}{a}$$

Since c is always smaller than a, the value of e will always be between 0 and 1 (for an ellipse that is not a circle).

Example

What is the eccentricity of the ellipse defined by.

$$9x^2 + 4y^2 + 36x - 8y + 4 = 0$$

Example

At its closest, the Moon is about 362,600 km away. At its farthest it is about 405,400 km away. What is the approximate eccentricity of the lunar orbit?