Name Solutions

Period______

Precalculus with Limits
Test Review
Parametric and Polar Equations

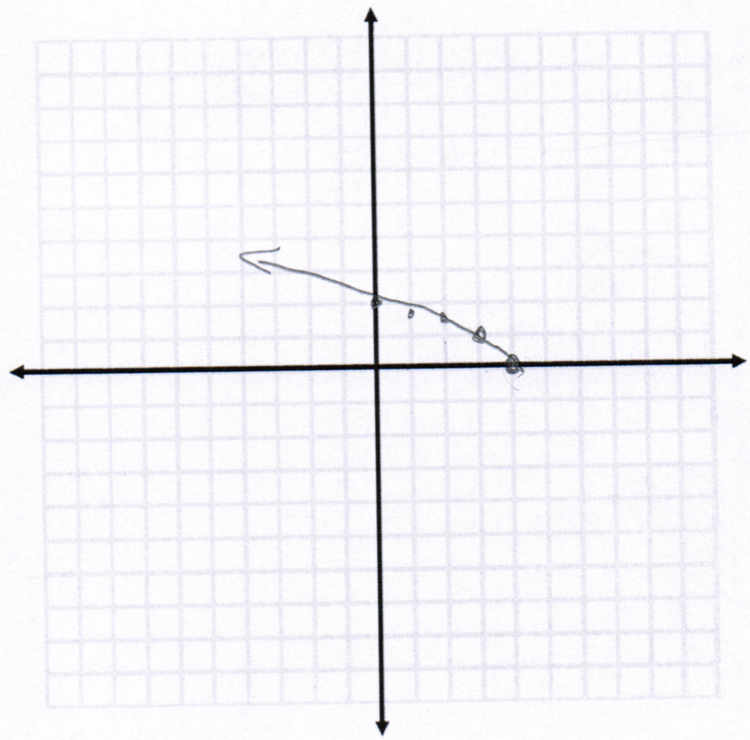
For each of the following parametric equations in #'s 1-3:

a) Graph by constructing a table of values for t, x, and y for the specified interval of t.

b) Obtain the rectangular equation by eliminating the parameter.

1)
$x(t) = 4 - t$
$y(t) = \sqrt{t}$
$0 \leq t \leq 4$

t	x	y
0	4	0
1	3	1
2	2	1.4
3	1	1.7
4	0	2



Rectangular equation $y = \sqrt{4-x}$

$x = 4 - t$

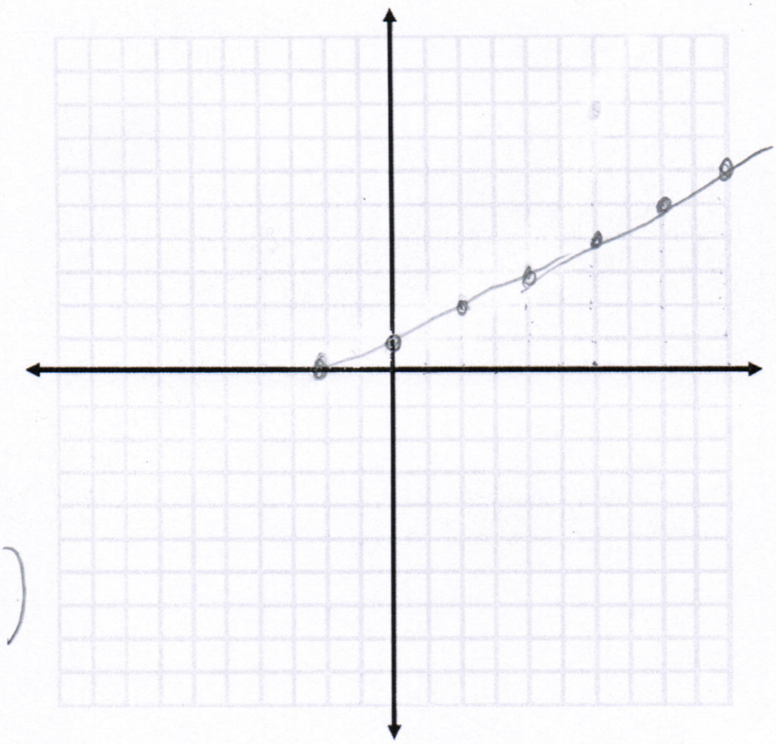
$t = 4 - x$

$y = \sqrt{4-x}$

2)
$x(t) = 4t - 2$
$y(t) = 2t$
$0 \le t \le 3$

t	x	y
0	-2	0
1/2	0	1
1	2	2
1.5	4	3
2	6	4
2.5	8	5
3	10	6

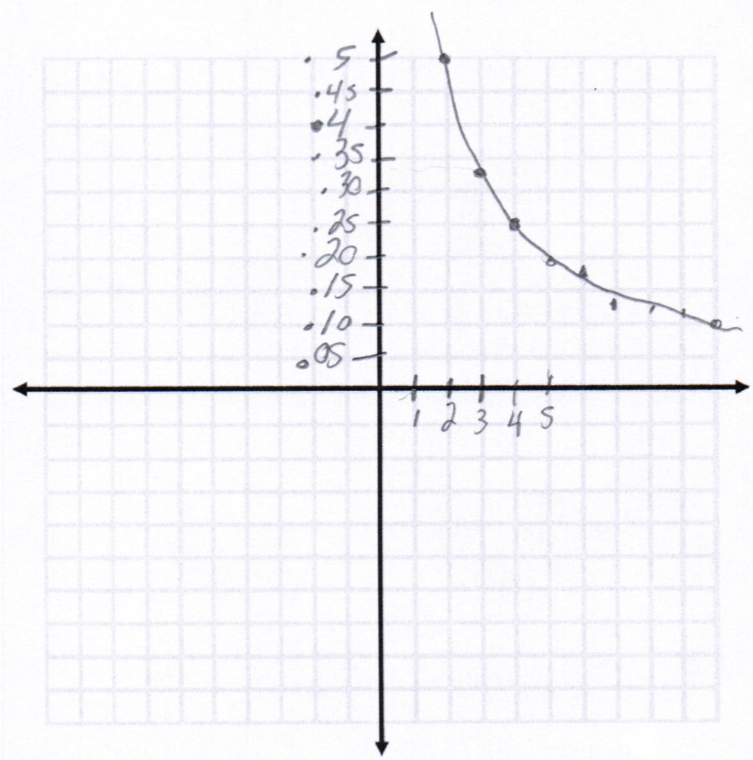
$x = 4t - 2$
$\frac{x+2}{4} = t$
$y = 2\left(\frac{x+2}{4}\right)$



Rectangular equation $y = \frac{1}{2}x + 1$

3)
$x(t) = t$
$y(t) = \frac{1}{t}$
$1 \le t \le 10$

t	x	y
1	1	1
2	2	1/2 .5
3	3	1/3 .33
4	4	1/4 .25
5	5	1/5 .2
6	6	1/6 .17
7	7	1/7 .14
8	8	1/8 .125
9	9	1/9 .11
10	10	1/10 .1



Rectangular equation $y = \frac{1}{x}$

4. The path of a projectile fired from ground level with an initial velocity of v_o feet per second at an angle α with the ground is given by the parametric equations

$$x(t) = (v_o \cos\alpha)t$$
$$y(t) = -16t^2 + (v_o \sin\alpha)t$$

a) Let $\alpha = 30°\ and\ v_0 = 20\ ft/s$. What is the horizontal distance traveled by the object?

$x = 20\cos(30°)t$
$y = -16t^2 + 20\sin(30°)t$

graph and trace until $y=0$

$t = .625$

10.83 ft.

b) Now let $\alpha = 60°\ and\ v_0 = 20\ ft/s$. What is the horizontal distance traveled by the object?

$x = 20\cos(60°)t$
$y = -16t^2 + 20\sin(60°)t$

10.83 ft. again

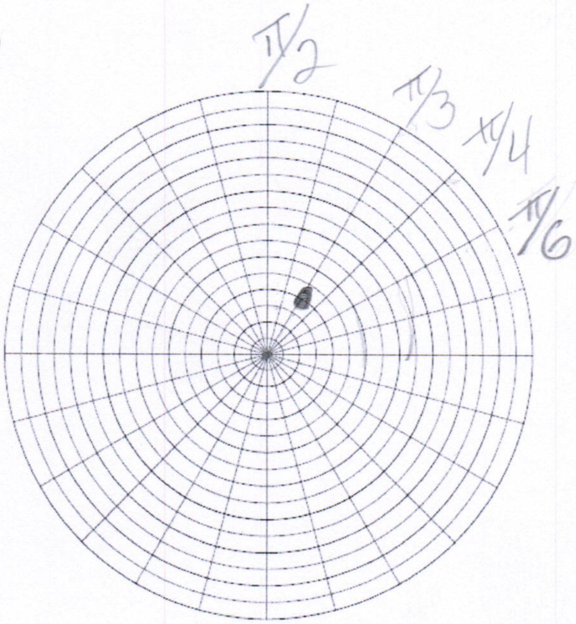
c) Can you make a conjecture about horizontal distance of a projectile and angle measure?

Complementary angles with same initial velocity result in the object landing in the same place.

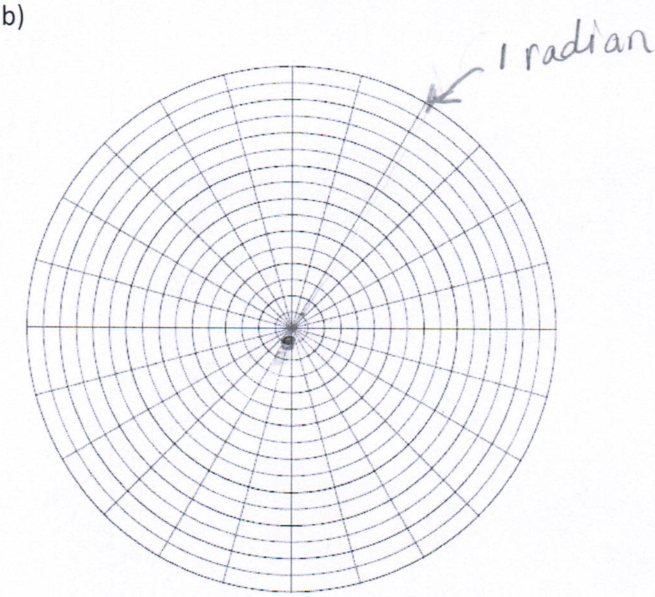
5. Plot the points whose polar coordinates are

a) $\left(4, \frac{\pi}{3}\right)$ b) $(-1,1)$ c) $\left(-2, -\frac{\pi}{2}\right)$ d) $\left(3, -\frac{\pi}{6}\right)$

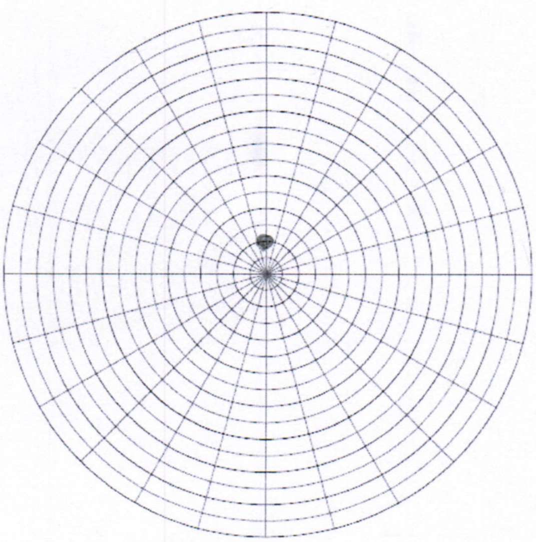
a)



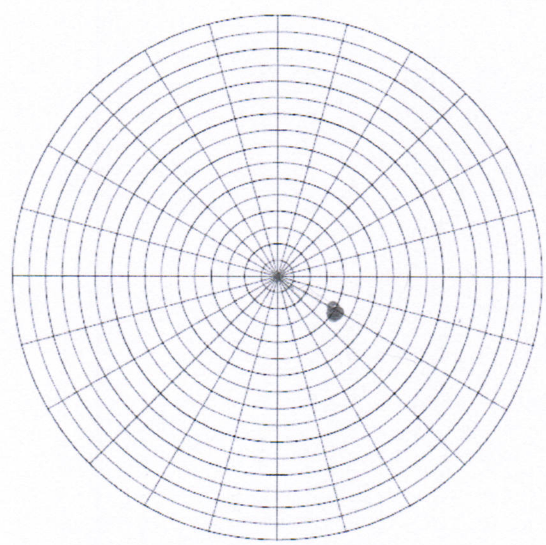
b)



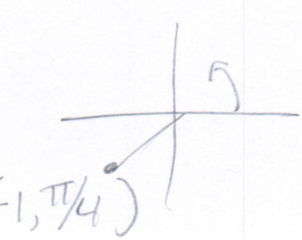
c)



d)



6. Consider the point given in polar coordinates $\left(-1, \frac{\pi}{4}\right)$. Give four other pairs of polar coordinates that represent the same point. Two must have a positive r value and two must have a negative r value.



$(1, 5\pi/4)$ $(1, -3\pi/4)$

$(-1, -7\pi/4)$ $(-1, \frac{\pi}{4}+2\pi)$

7. Convert the following points from polar to rectangular coordinates.

a) $\left(3\sqrt{2}, \frac{7\pi}{2}\right)$

$X = r\cos\theta$
$\cancel{X} = 3\sqrt{2}\cos(7\pi/2)$
$X = 0$
$y = r\sin\theta$
$y = 3\sqrt{2}(-1)$
$(0, -3\sqrt{2})$

b) $\left(\sqrt{2}, -\frac{\pi}{3}\right)$

$x = \sqrt{2}\cos(-\pi/3)$
$x = \sqrt{2} \cdot \frac{1}{2}$
$y = \sqrt{2}\sin(-\pi/3)$
$y = \sqrt{2}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{6}}{2}$
$(\sqrt{2}/2, -\sqrt{6}/2)$

c) $\left(-3, -\frac{\pi}{6}\right)$

$X = -3\cos(-\pi/6)$
$X = -3 \cdot \frac{\sqrt{3}}{2}$
$y = -3\sin(-\pi/6)$
$y = -3(-1/2) = 3/2$
$\left(-\frac{3\sqrt{3}}{2}, 3/2\right)$

8. Convert the following points from rectangular to polar coordinates.

a) $(3\sqrt{3}, 3)$

$\tan\theta = \frac{3}{3\sqrt{3}}$
$\tan\theta = \frac{1}{\sqrt{3}}$
$\theta = \pi/6$
$r^2 = 9 \cdot 3 + 9$
$r = 6$
$(6, \pi/6)$

b) $(3, -4)$

$\tan\theta = \frac{-4}{3}$
$\theta = -.93$
$r = 5$
$(-5, -.93)$

c) $(-2\sqrt{3}, 2)$

$\tan\theta = \frac{2}{-2\sqrt{3}} = \frac{-1}{\sqrt{3}}$
$\theta = 5\pi/6$
$r^2 = 4 \cdot 3 + 4$
$r = 4$
$(4, 5\pi/6)$

9. Find the polar equation for $x^2 + y^2 = 4$.

$r = 4$

10. Find the polar equation for $x^2 = -16y + 2$

$(r\cos\theta)^2 = -16(r\sin\theta) + 2$

11. Find the rectangular equation for $r = 4\sin\theta$.

$r^2 = 16\frac{x^2}{r^2}$

$x^2 + y^2 = \frac{16x^2}{x^2+y^2}$ or $\to (x^2+y^2)^2 = 16x^2$

or

$x^2 + y^2 = 4x$

12. Find the rectangular equation for $r = \frac{6}{4-\cos\theta}$.

$4r - r\cos\theta = 6$
$4r - x = 6$
$4r = 6 + x$
$16r^2 = 36 + 12x + x^2$
$16(x^2+y^2) = 36 + 12x + x^2$

13. Find the rectangular equation for $r = 6$.

$r^2 = 36$
$x^2 + y^2 = 36$

14. Find the rectangular equation for $\theta = \frac{2\pi}{3}$.

$\tan\theta = \tan\frac{2\pi}{3}$

$\frac{y}{x} = -\sqrt{3}$

$y = -\sqrt{3}x$

List all symmetries for each of the following polar graphs. Perform an algebraic test for each type of symmetry. Remember that a polar graph can algebraically fail test but still pass graphically.

15. $r = 4 - 3\cos(\theta)$

polar axis only

16. $r = \sqrt{2} - \sqrt{2}\sin\theta$

$\theta = \pi/2$ only

17. $r = 3\sin(3\theta)$

$\theta = \pi/2$ only

18. $r^2 = -9\cos(2\theta)$

$\theta = \pi/2$ and pole, and polar axis

19. $r = e^{\theta}$

none

20. ~~15~~. Prove that $r = a\sin\theta + b\cos\theta$ represents a circle and find its radius and center.

This problem is for fun. It won't be on test.

multiply both sides by r

$r^2 = ar\sin\theta + br\cos\theta$

$x^2 + y^2 = ay + bx$

$x^2 - bx + y^2 - ay = 0$

$\left(x^2 - bx + \frac{b^2}{4}\right) + \left(y^2 - ay + \frac{a^2}{4}\right) = \frac{b^2}{4} + \frac{a^2}{4}$

$\left(x - \frac{b}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \frac{b^2 + a^2}{4}$

center $\left(\frac{b}{2}, \frac{a}{2}\right)$ radius $= \frac{\sqrt{b^2 + a^2}}{2}$

Graph the following. Use polar graph paper.

21. ~~16~~. $r = 2 - 3\sin\theta$ (limacon)

22. ~~17~~. $r = e^{\theta}$ (logarithmic spiral)

See final page

23. ~~18~~. $r^2 = -9\cos(2\theta)$ (lemniscate)

24 ~~19~~. $r = 5 - 5\sin\theta$

25. ~~20~~. $r = -4\sec\theta$

26. ~~21~~. $\theta = -\frac{\pi}{3}$

27. ~~22~~. Determine where the polar graphs intersect.

$r = 3\sqrt{3}\cos\theta$
$r = 3\sin\theta$

$3\sqrt{3}\cos\theta = 3\sin\theta$

$\sqrt{3}\cos\theta = \sin\theta$

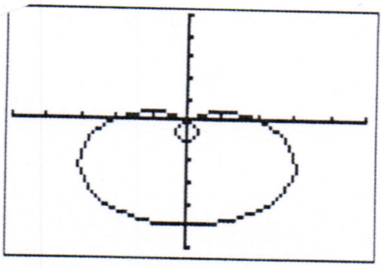
$\sqrt{3} = \frac{\sin\theta}{\cos\theta}$

$\sqrt{3} = \tan\theta$

$\theta = \pi/3, 4\pi/3$

28. Also a fun problem.

21.



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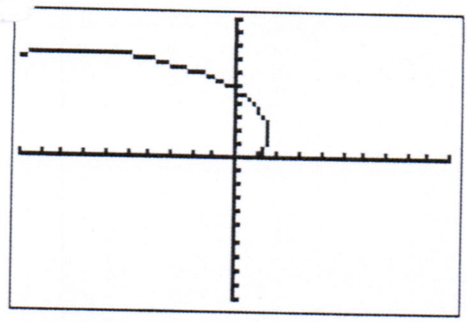

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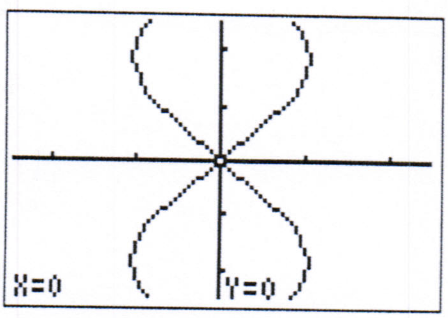

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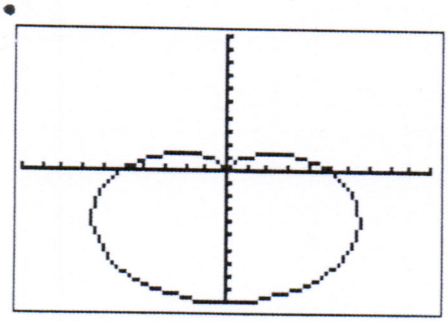

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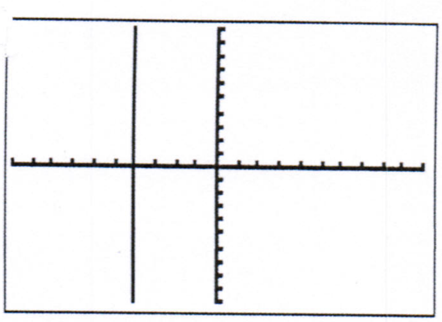

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