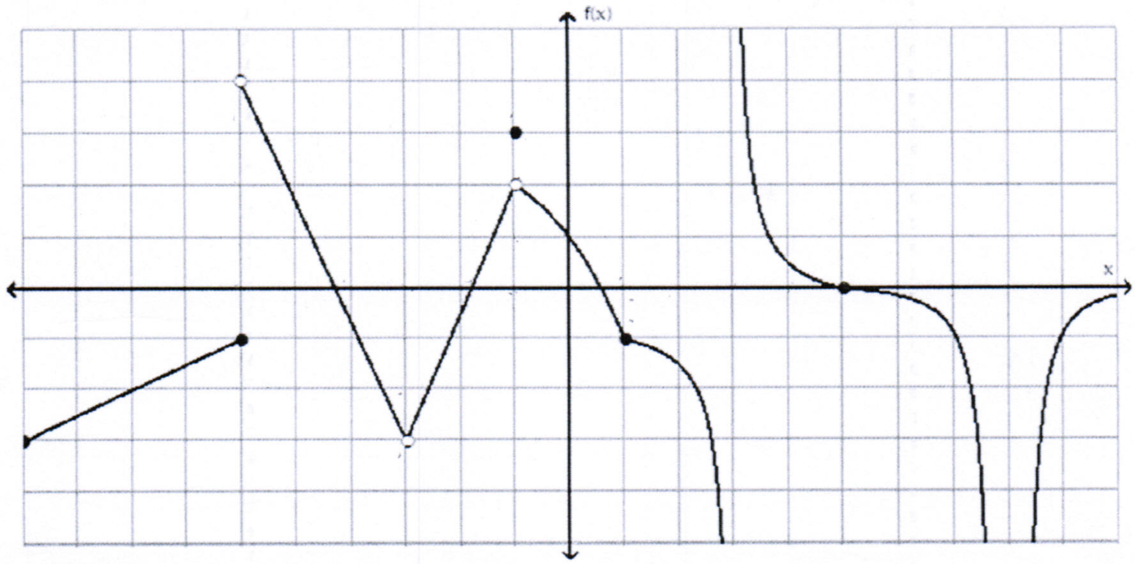
Precalculus with Limits
Test Review
Limits

Name Solutions
Period______

1. Use the graph to determine the following.



a)
$\lim_{x\to -6} f(x) =$ DNE

b)
$\lim_{x\to -6^+} f(x) =$ 4

c)
$\lim_{x\to -3} f(x) =$ -3

d)
$f(-3) =$ undefined

e)
$\lim_{x\to -1} f(x) =$ 2

f)
$f(-1) =$ 3

g)
$\lim_{x\to 1} f(x) =$ -1

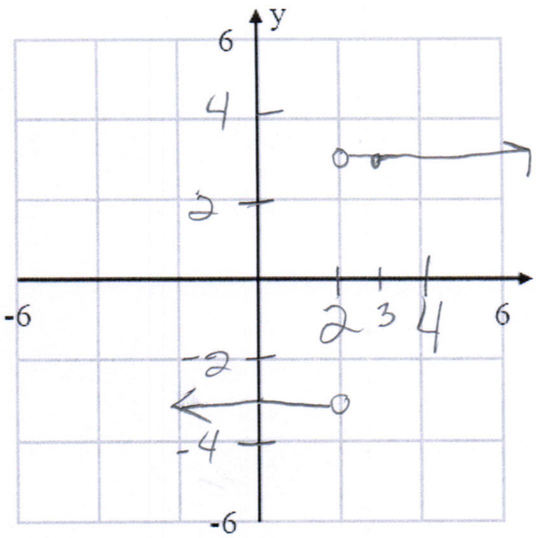
h)
$\lim_{x\to 5} f(x) =$ 0

i)
$\lim_{x\to 8} f(x) =$ $-\infty$
or
DNE

j)
$\lim_{x\to 0} f(x) =$ 1

2. Let $f(x) = \frac{3x-6}{|x-2|}$.

Draw the function on the grid provided. (Use your calculator to generate the graph.) Then determine the following.



a) $\lim_{x \to 3} f(x) = 3$

b) $\lim_{x \to 2^-} f(x) = -3$

c) $\lim_{x \to 2^+} f(x) = 3$

d) $\lim_{x \to 2} f(x) =$ DNE

e) $f(2) =$ undefined

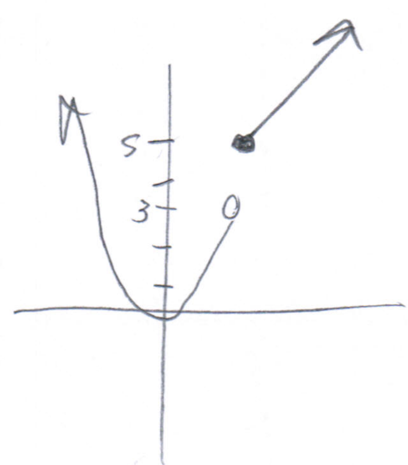
f) $\lim_{x \to \infty} f(x) = 3$

3. Let

$$f(x) = \begin{cases} 4x^2 - 1 & \text{if } x < 1 \\ 3x + 2 & \text{if } x \geq 1 \end{cases}$$

Use the graphing calculator to examine the graph of f(x) and find

$\lim_{x \to 1^+} f(x) = 5$

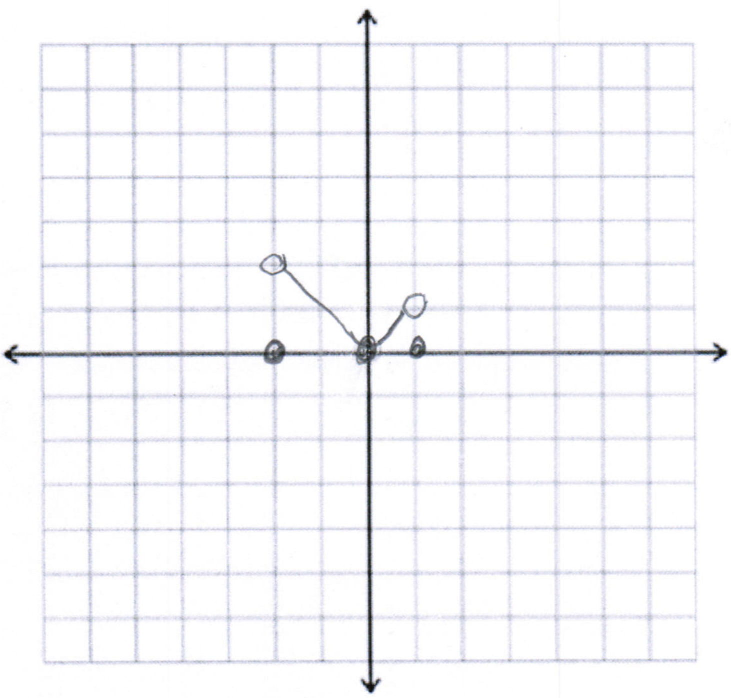


4. On the grid below, sketch a graph of a function with the following characteristics.

i. the domain is [-2,1]

ii. $f(-2) = f(0) = f(1) = 0$

iii. $\lim\limits_{x\to -2^+} f(x) = 2$, $\lim\limits_{x\to 0} f(x) = 0$, $\lim\limits_{x\to 1^-} f(x) = 1$



Find the following limits algebraically. Show as much work as possible. (note: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$)

5.

$$\lim_{x\to 2} \frac{3x^2 - x + 1}{1 + 7x}$$

$$= \frac{3(2)^2 - (2) + 1}{1 + 7\cdot 2} = \frac{11}{1+14} = \boxed{\frac{11}{15}}$$

6.

$$\lim_{x\to -3} \frac{x^3 + 27}{x + 3} = \lim_{x\to -3} \frac{\cancel{(x+3)}(x^2 - 3x + 9)}{\cancel{x+3}}$$

$$\lim_{x\to -3} x^2 - 3x + 9 = (-3)^2 - 3(-3) + 9$$

$$= 9 + 9 + 9 = \boxed{27}$$

7.

$$\lim_{x\to -\infty} \frac{3x^2 - 6x + 1}{-6x^2 + x + 2}$$

$$-\frac{3}{6} = \boxed{-\frac{1}{2}}$$

8.

$$\lim_{x\to \infty} \sqrt[3]{\frac{3 - 128x^4}{2x^4 + x^3 - x + 2}} = \sqrt[3]{\lim_{x\to\infty} \frac{3 - 128x^4}{2x^4 + x^3 - x + 2}}$$

$$= \sqrt[3]{\frac{-128}{2}} = \sqrt[3]{-64} = \boxed{-4}$$

9.

$$\lim_{x\to 0^+} \frac{\csc 2x}{x} = \lim_{x\to 0^+} \frac{1}{\sin 2x} \cdot \frac{1}{x}$$

$$= \lim_{x\to 0^+} \frac{2x}{\sin 2x} \cdot \frac{1}{x} \cdot \frac{1}{2x} = 1 \cdot \frac{1}{0}$$

DNE

10.

$$\lim_{\theta\to 0} \frac{\cot(\theta)\sin(\pi\theta)}{4\sec(\theta)} = \lim_{\theta\to 0} \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin(\pi\theta)}{\frac{1}{\cos\theta}} \cdot \frac{1}{4}$$

$$\lim_{\theta\to 0} \frac{\cos^2\theta}{4} \cdot \frac{\theta}{\sin\theta} \cdot \frac{\sin\pi\theta}{\pi\theta} \cdot \pi$$

$$\frac{1}{4} \cdot 1 \cdot 1 \cdot \pi$$

$\pi/4$

11.

$$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} \cdot \frac{(\sqrt{x}+5)}{\sqrt{x}+5}$$

$$\lim_{x\to 25} \frac{\cancel{x-25}}{\cancel{(x-25)}(\sqrt{x}+5)}$$

$$\lim_{x\to 25} \frac{1}{\sqrt{x}+5} = \frac{1}{\sqrt{25}+5} = \boxed{\frac{1}{10}}$$

12.

$$\lim_{\theta\to 0} \frac{\cos\theta\tan\theta}{\theta}$$

$$\lim_{\theta\to 0} \frac{\cancel{\cos\theta}}{1} \cdot \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{1}{\theta}$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

13.

$$\lim_{x\to 8} \frac{x+5}{x-8}$$

∞ or DNE

14.

$$\lim_{x\to 0} \frac{x+5}{x-8} = \boxed{\frac{5}{-8}}$$

15. Use your graphing calculator to determine the following limit.

$$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e \approx 2.718281828459045\ldots$$

16. Let $\lim_{x \to a} f(x) = -4$ and $\lim_{x \to a} g(x) = 8$. Use properties of limits to determine the following.

a) $\lim_{x \to a} \sqrt{f^2(x) + g^2(x)}$

$\sqrt{(-4)^2 + 8^2}$

$\sqrt{16 + 64}$

$\sqrt{80}$

b) $\lim_{x \to a} \frac{f(x) - 3g(x)}{5f(x) + 2g(x)} = \frac{-4 - 3(8)}{5(-4) + 2(8)}$

$= \frac{-4-24}{-20+16} = \frac{-28}{-4} = \boxed{7}$

c) $\lim_{x \to a} \sqrt[3]{g(x)} \cdot [f(x) + 3]$

$\sqrt[3]{8}\,[-4+3]$

$2(-1) = -2$

d) $\lim_{x \to a} [g(x) - f(x)]^2$

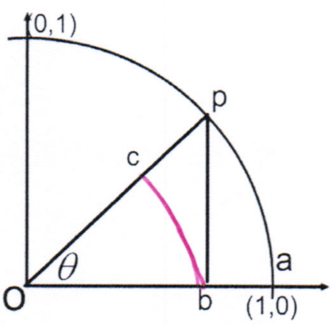
$[8 - -4]^2$

12^2

144

17.

Prove: $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta}$



see notes!