Precalculus with Limits
Practice
Optimization

Name solutions
Date______
Period______

1.

Find two numbers whose sum is 12 if the product of the square of one number with the square root of the other number is to be a maximum.

$x+y=12$

$S=x^2\sqrt{y}$

$S=x^2\sqrt{12-x}$

$S'=2x\sqrt{12-x}+\tfrac{1}{2}(12-x)^{-1/2}(-1)x^2$

$0=2x\sqrt{12-x}-\dfrac{x^2}{2\sqrt{12-x}}$

$2x\sqrt{12-x}=\dfrac{x^2}{2\sqrt{12-x}}$

$4x(12-x)=x^2$

$48x-4x^2=x^2$

$48x-5x^2=0$

$x(48-5x)=0$

$x=0 \quad 48-5x=0$

$\boxed{x=48/5}$

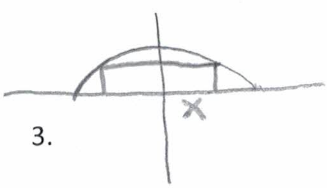
$\boxed{y=2.4}$

2.

Find the area of the largest rectangle, which may be inscribed under the ellipse $x^2+4y^2=4$ if one side of the rectangle is on the x-axis, while the vertices of the opposite side are on the ellipse.

$y^2=\dfrac{4-x^2}{4}$

$y=\sqrt{\dfrac{4-x^2}{4}}$



$A=\cancel{2}x\left(\dfrac{\sqrt{4-x^2}}{\cancel{2}}\right)$

$A=x\sqrt{4-x^2}$

$A'=1\cdot\sqrt{4-x^2}+\tfrac{1}{\cancel{2}}(4-x^2)^{-1/2}(-\cancel{2}x)x$

$0=\sqrt{4-x^2}-\dfrac{x^2}{\sqrt{4-x^2}}$

$\sqrt{4-x^2}=\dfrac{x^2}{\sqrt{4-x^2}}$

$4-x^2=x^2$

$4=2x^2$

$2=x^2$

$x=\sqrt{2}$

So Area $=2$

3.

Find the dimensions of the largest rectangular box if the length of the base of the box is to be three times the width, and the total surface area of the box is to be 200 square inches.

$SA=200=2hl+2wl+2wh$

$l=3w$ so $200=2h(3w)+2w(3w)+2wh$

$200=6wh+6w^2+2wh$

$200=8wh+6w^2$

$200-6w^2=8wh$

$\dfrac{200-6w^2}{8w}=h$

$V=lwh=3w\cdot\cancel{w}\left(\dfrac{200-6w^2}{8\cancel{w}}\right)$

$V=\dfrac{3w(200-6w^2)}{8}=\dfrac{600w-18w^3}{8}$

$V=75w-\tfrac{18}{8}w^3$

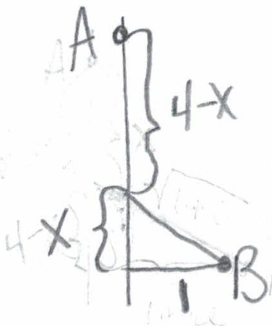
$V'=75-\tfrac{54}{8}w^2=0$

$75=\tfrac{54}{8}w^2 \quad w^2=11.\overline{1}$

So $w=10/3$ $\quad l=10 \quad h=5$

4.

A computer company wishes to run a cable from point A which is located on the shore of a river to a point B which is located on an island 4 miles downstream and 1 mile offshore. The costs of running the cable are \$300 per mile on land and \$500 per mile in the water. Find the length of the cable, which should be run on land if the total cost of the project is to be as small as possible.



$$\text{Cost} = C = 300(4-x) + 500\sqrt{1+x^2} = 1200 - 300x + 500\sqrt{1+x^2}$$

$$C' = -300 + 500\left(\tfrac{1}{2}\right)(1+x^2)^{-1/2}\, 2x = 0$$

$$\frac{500x}{\sqrt{1+x^2}} = 300$$

$$500x = 300\sqrt{1+x^2}$$

$$5x = 3\sqrt{1+x^2}$$

$$25x^2 = 9(1+x^2)$$

$$25x^2 = 9 + 9x^2$$

$$16x^2 = 9$$

$$x^2 = 9/16$$

$$x = 3/4$$

and so

$$4 - 3/4 = 3.25$$

5.

A man lauches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. He could proceed in any of three ways:

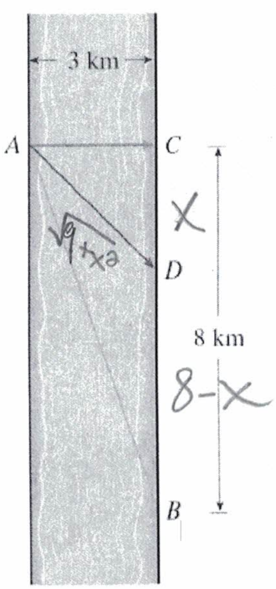
1. Row his boat directly across the river to point C and then run to B.

2. Row directly to B.

3. Row to some point D between C and B and then run to B.

If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as possible?

$$\text{Total time} = T = t_{row} + t_{run}$$



$$\sqrt{9+x^2} = 6\, t_{row} \qquad 8 - x = 8\, t_{run}$$

$$\frac{\sqrt{9+x^2}}{6} = t_{row} \qquad \frac{8-x}{8} = t_{run}$$

$$T = \frac{1}{6}\sqrt{9+x^2} + 1 - \frac{1}{8}x$$

$$T' = \frac{1}{6}\cdot\frac{1}{2}(9+x^2)^{-1/2}\, 2x - \frac{1}{8} = 0$$

$$T' = \frac{x}{6\sqrt{9+x^2}} = \frac{1}{8}$$

$$8x = 6\sqrt{9+x^2}$$

$$64x^2 = 36(9+x^2)$$

$$64x^2 = 324 + 36x^2$$

$$28x^2 = 324$$

$$x^2 = 324/28$$

$$x = 3.40$$