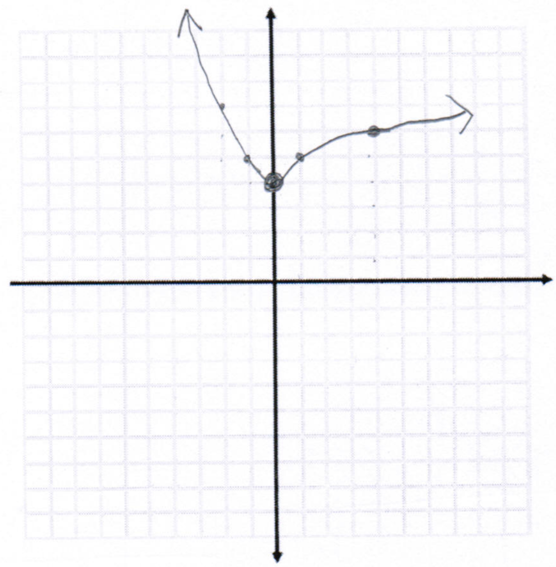
Precalculus with Limits
Review
Chapter 1

Name Solutions

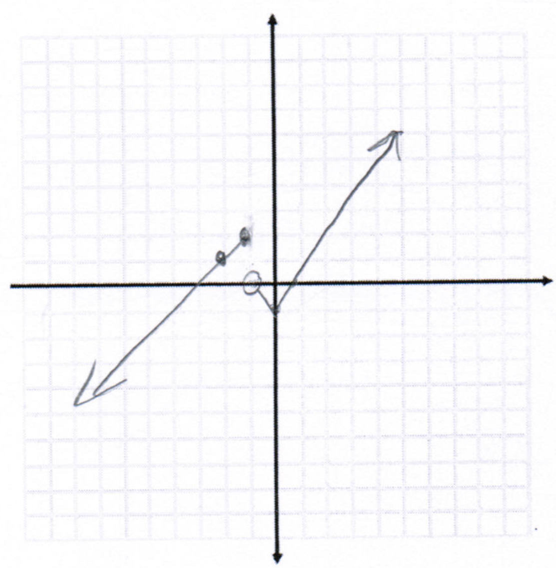
Period________

Graph the following piecewise functions.

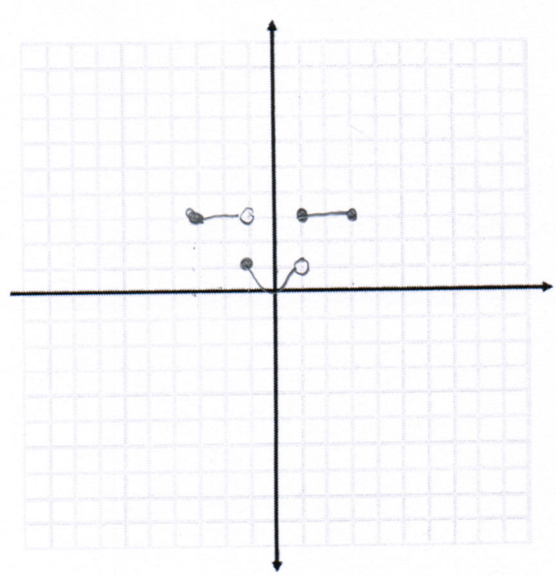
1. $f(x)=\begin{cases} x^2+4, x<0 \\ \sqrt{x}+4, x\geq 0 \end{cases}$



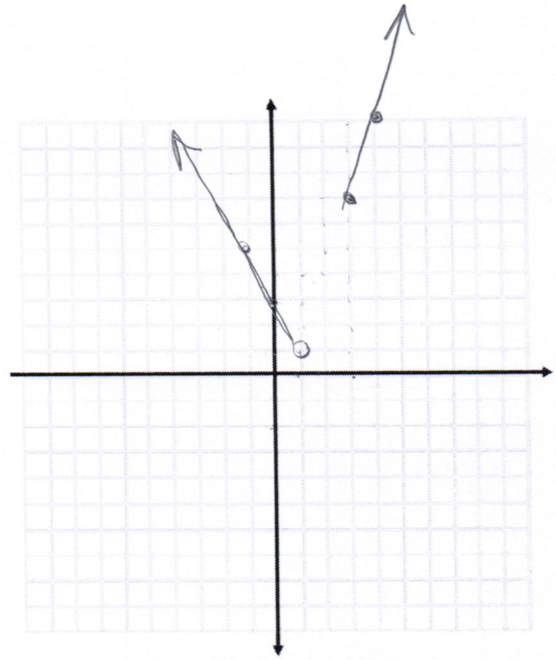
2. $f(x)=\begin{cases} |x|-1, x>-1 \\ x+3, x\leq -1 \end{cases}$



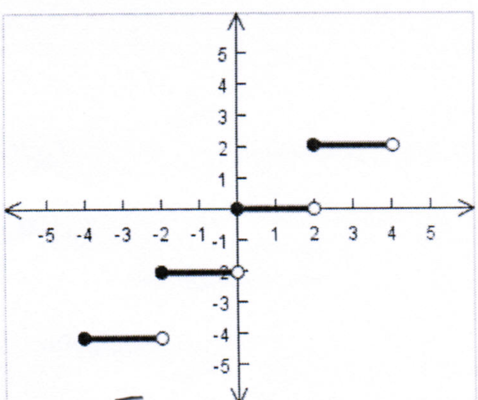
3. $f(x)=\begin{cases} 3, for -3\leq x<-1 \\ x^2, for -1\leq x<1 \\ 3, for 1\leq x\leq 3 \end{cases}$



4. $f(x) = \begin{cases} -2x+3 & for \ \ x<1 \\ 3x-2 & for \ \ x \geq 3 \end{cases}$

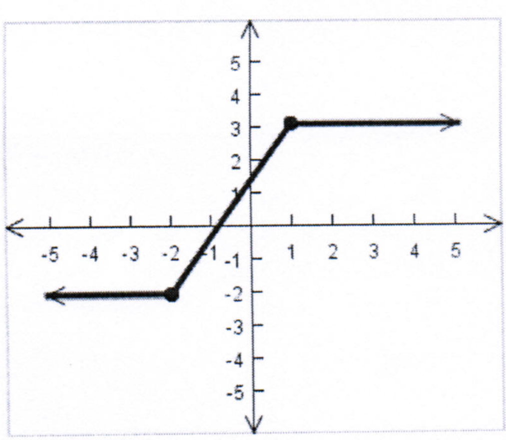


5. Write the equation of a greatest integer function for the graph shown.



Y = $2[\![\tfrac{1}{2}x]\!]$

6. Write the equation of a piecewise function for the graph shown.



$(1,3)\ (-2,-2)$

$m = \dfrac{-2-3}{-2-1} = \dfrac{-5}{-3} = \dfrac{5}{3}$

$y-3 = \frac{5}{3}(x-1) \quad y = \frac{5}{3}x - \frac{5}{3} + \frac{3}{1}$

$y = \frac{5}{3}x + \frac{4}{3}$

$f(x) = \begin{cases} -2 & x \leq -2 \\ \frac{5}{3}x + \frac{4}{3} & -2 < x \leq 1 \\ 3 & x > 1 \end{cases}$

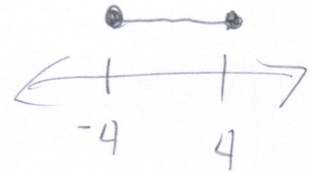
State the domain for the following functions.

7. $f(x) = \frac{x+9}{x^2-7x}$

$= \frac{x+9}{x(x-7)}$

$(-\infty, 0) \cup (0, 7) \cup (7, \infty)$

8. $g(x) = \sqrt{16 - x^2}$

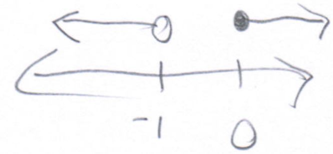


$[-4, 4]$

9. $f(x) = \frac{1}{\sqrt{x+8}}$

$(-8, \infty)$

10. $h(x) = \sqrt{\frac{x}{x+1}}$



$(-\infty, -1) \cup [0, \infty)$

11. Consider the following functions.

$f(x) = x^2$

$g(x) = \sqrt{\frac{1}{2}x - 3}$

$\frac{1}{2}x - 3 \geq 0 \quad \frac{1}{2}x \geq 3$

$x \geq 6$

a) State the domain of f(x).

$(-\infty, \infty)$

b) State the domain of g(x).

$[6, \infty)$

c) What is f(g(x))? Do not simplify the result. Now graph f(g(x)) on your calculator while still not simplifying the equation.

$f(g(x)) = \left(\sqrt{\frac{1}{2}x - 3}\right)^2$

d) NOW simplify the result from part c) and graph it on the calculator.

$y = \frac{1}{2}x - 3$

e) What can you conclude from this exploration?

12. Let $h(x) = \left(\frac{1-x^2}{1+x^2}\right)^5$

State two functions, f(x) and g(x), such that f(g(x))=h(x).

$f(x) = \left(\frac{1-x}{1+x}\right)^5$

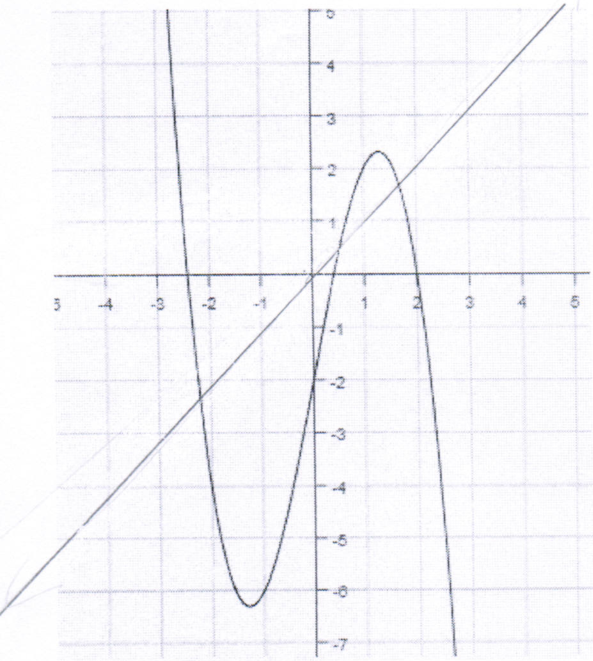
$g(x) = x^2$

or

$f(x) = x^5$

$g(x) = \frac{1-x^2}{1+x^2}$

13. Draw the graph of the inverse for the following function.



14. Explain under what condition(s) an inverse of a function will itself be a function. Why?

If the original function is one-to-one (i.e. passes horizontal line test), then its inverse will also be a function. One-to-one implies 1 x for each y, so the inverse will have 1 y for each x.

15. Students often make the mistake of thinking that $f^{-1}(x)$ means to take the reciprocal of f(x). For example, consider the function $f(x) = x^3 - 2$. Someone might make the mistake and think that $f^{-1}(x) = \frac{1}{f(x)} = \frac{1}{x^3-2}$.

The correct answer is $f^{-1}(x) = \sqrt[3]{x-2}$.

Search your mind for a function which satisfies $f^{-1}(x) = \frac{1}{f(x)}$. That is, find a function whose inverse also happens to be its reciprocal.

F(x)= answers vary

16. Let $g(x) = \frac{5x+2}{3-x}$.

a) What is $g^{-1}(x)$?

$x = \frac{5y+2}{3-y}$

$x(3-y) = 5y+2$

$3x - xy = 5y+2$

$3x-2 = 5y+xy$

$3x-2 = y(5+x)$

$\boxed{y = g^{-1}(x) = \frac{3x-2}{5+x}}$

b) Verify that $g(g^{-1}(x)) = x$.

$g(g^{-1}(x)) = \dfrac{5\left(\frac{3x-2}{5+x}\right)+2}{3-\left(\frac{3x-2}{5+x}\right)} = \dfrac{\frac{15x-10}{5+x} + \frac{2(5+x)}{5+x}}{\frac{3(5+x)}{5+x} - \frac{3x-2}{5+x}} = \dfrac{\frac{15x-10+10+2x}{5+x}}{\frac{15+3x-3x+2}{5+x}}$

$= \frac{17x}{5+x} \cdot \frac{5+x}{17} = x$ ✓

c) Verfiy that $g^{-1}(g(x)) = x$.

$g^{-1}(g(x)) = \dfrac{3\left(\frac{5x+2}{3-x}\right)-2}{5+\frac{5x+2}{3-x}} = \dfrac{\frac{15x+6}{3-x} - \frac{2(3-x)}{3-x}}{\frac{5(3-x)}{3-x} + \frac{5x+2}{3-x}} = \dfrac{\frac{15x+6-6+2x}{3-x}}{\frac{15-5x+5x+2}{3-x}}$

$= \dfrac{\frac{17x}{3-x}}{\frac{17}{3-x}} = \frac{\cancel{17}x}{\cancel{3-x}} \cdot \frac{\cancel{3-x}}{\cancel{17}} = x$

d) Complete the following tables of values for $g(x)$ and $g^{-1}(x)$.

oops!

x	~~f(x)~~ g(x)
-3	-13/6
1	7/2
2	12
3	undefined

oops!

x	~~$f^{-1}(x)$~~ $g^{-1}(x)$
-13/6	-3
7/2	1
12	2
not possible	3