Precalculus with Limits
Practice Test
Exponentials and Logarithms

Name Solutions

Period ____

Find $f^{-1}(x)$ for #'s 1-4.

1. $f(x) = \sqrt[3]{3x-10}$

$x = \sqrt[3]{3y-10}$

$\frac{x^3+10}{3} = y$

2. $f(x) = \frac{1}{1+x}$ $x \geq 0$

$x = \frac{1}{1+y}$

$x(1+y) = 1$

$x + xy = 1$

$xy = 1 - x$

$y = \frac{1-x}{x}$

3. $f(x) = 3^x$

$f^{-1}(x) = \log_3 x$

4. $f(x) = \ln(x)$

$f^{-1}(x) = e^x$

Expand

5. $\ln\sqrt{x^3(x+2)}$

$\frac{1}{2}(3\ln x + \ln(x+2))$

6. $\log_b \frac{\sqrt{x}y^4}{z^4}$

$\frac{1}{2}\log_b x + 4\log_b y - 4\log_b z$

7. $\log_c \sqrt{\frac{x}{x+7}}$

$\frac{1}{2}\log_c x - \frac{1}{2}\log_c(x+7)$

8. $\log_3 \frac{x^3y^2}{z}$

$3\log_3 x + 2\log_3 y - \log_3 z$

9. $\log_3 7t$

$\log_3 7 + \log_3 t$

Condense

10. $2[\ln x - \ln(x+1) - \ln(x-1)]$

$\ln\left(\left(\frac{x}{x^2-1}\right)^2\right)$

12. $2\ln 8 + 5\ln z$

$\ln(64z^5)$

13. $2\log_5 x + 7\log_5 x$

$\log_5(x^9)$

14. $\log_b \frac{x+y}{z} - \log_b \frac{1}{x+y}$

$\log_b\left(\frac{x+y}{z} \div \frac{1}{x+y}\right)$

$\log_b\left(\frac{(x+y)^2}{z}\right)$

15. Show that $ln\left(\frac{1}{5}\right) = -\ln(5)$

$\ln(1) - \ln(5) = -\ln(5)$

$0 - \ln(5) = -\ln(5)$

Solve

16. $6^{5x} = 3000$

$5x \ln 6 = \ln 3000$

$x = \frac{\ln 3000}{5\ln 6}$

$x \approx .894$

17. $\log_4 x - \log_4(x-1) = \frac{1}{2}$

$\log_4\left(\frac{x}{x-1}\right) = 1/2$

$4^{1/2} = \frac{x}{x-1}$

$2(x-1) = x$

$2x - 2 = x$

$x - 2 = 0$

$x = 2$

18. $\log_3 x + \log_3(x-8) = 2$

$\log_3(x^2 - 8x) = 2$

$9 = x^2 - 8x$

$0 = x^2 - 8x - 9$

$(x-9)(x+1) = 0$

$x = 9$ $x = -1$

19. $4^{-3t} = 0.10$

$-3t \ln 4 = \ln(0.10)$

$t = \frac{\ln(0.10)}{-3\ln 4} \approx .554$

20. $\log_5(2x-3) = 2$

$25 = 2x - 3$

$28 = 2x$

$x = 14$

21. $\log_{3/4} \frac{4}{3} = x$

$\frac{3}{4}^x = \frac{4}{3}$

$x = -1$

22. $3^{2x+1} = 27$

so $2x+1=3$

$3^{2x+1} = 3^3$

$2x=2$

$x=1$

23. Exponential functions cannot have a base of 1 because

a) it can create imaginary numbers
b) the graph would show exponential decay
c) the graph would be a horizontal line
d) the graph would show exponential growth

24. Exponential function cannot have a negative number as a base because

a) imaginary numbers could arise
b) the graph would show oscillations between negative and positive numbers
c) you cannot take the log of a negative number
d) a and b

25. $5^{\log_5 7} = 7$ because

a) $f(f^{-1}(x)) = x$
b) logs and exponentials of the same base are inverse operations
c) a composition of a function and its inverse simplifies to the original input value
d) all answers choices are correct

26. A logarithm can be thought of as

a) the reciprocal of an exponential function.
b) unit of measure for money
c) a reflection of an exponential function over the line y = x.
d) division by an exponent

27. If you invest $2000 in a savings account at 5% annual interest compounded quarterly, how long will it take to accumulate a total of $10,000? Solve algebraically.

$10,000 = 2000\left(1+\frac{.05}{4}\right)^{4t}$

$5 = \left(1+\frac{.05}{4}\right)^{4t}$

$\ln 5 = 4t \ln\left(1+\frac{.05}{4}\right)$

$t \approx 32.4$ yrs.

28. Dino, a student in Alg. II-Trig typed "ln(0)" into the calculator and hit enter. He started to cry when an error message appeared on the screen. Why did he get an error message?

$\ln(0) = x$

$e^x = 0$

there is no value x that will transform e into 0.

29. If you invest $1000 in a savings account at 6% annual interest compounded continuously, how long will it take for your money to double?

$2 = e^{.06t}$

$\ln 2 = .06t$

11.55 yrs.

30. Let f be a logarithmic function with base b (b>1). What is the range of f? Explain how you know.

$f(x) = \log_b x$ ← range is $(-\infty, \infty)$

$f^{-1}(x) = b^x$ ← Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ so switch domain and range

31. Use the formula for discrete compound interest (i.e. $A = P\left(1+\frac{r}{n}\right)^{nt}$) to derive the formula for continuous compounding.

See notes

32. What is a logarithm?

inverse of exponential function. or exponent necessary

33. The concentration of hydrogen ions in a substance is denoted H^+.

The pH level of a substance is calculated with the formula

$$pH = -\log(H^+)$$

Find the concentrations of hydrogen ions in the following foods. Write your answers in scientific notation.

Food	pH	Concentrations
Apple-Golden Delicious	3.6	$10^{-3.6} = 2.5 \times 10^{-4}$
Cauliflower	5.6	$10^{-5.6} = 2.5 \times 10^{-6}$
Tofu	7.2	$10^{-7.2} = 6.3 \times 10^{-8}$

$3.6 = -\log(H^+)$

$-3.6 = \log(H^+)$

$10^{-3.6} = H^+$

$5.6 = -\log(H^+)$

$-5.6 = \log(H^+)$

$10^{-5.6} = H^+$

$-7.2 = \log H^+$

$10^{-7.2} = H^+$

34. Decibel levels are calculated with the formula $db = 10\log\left(\frac{I}{I_0}\right)$. Where I_0 represents the threshold of hearing, $I_0 = 10^{-12}\ W/m^2$.

a) The intensity of noise created by a chainsaw at a distance of 1 meter is about $10^{-1}\ W/m^2$. How many decibels is this sound?

$dB = 10\log\left(\frac{10^{-1}}{10^{-12}}\right) = 10\log\left(\frac{10^{12}}{10^{1}}\right) = 10\log(10^{11}) = 10 \cdot 11 = 110dB$

b) The rustling of leaves has a sound level of about 10 decibels. Determine the intensity of this sound.

$10 = 10\log\left(\frac{I}{10^{-12}}\right) \quad 1 = \log\left(\frac{I}{10^{-12}}\right) \quad 10^1 = \frac{I}{10^{-12}} \quad I = 10^{-11}$

35. A sample of 500 grams of lead-210 decays according to the function $y = y_0 e^{-0.032t}$.

a) How much lead will be left in the sample after 20 years? Round to the nearest tenth.

$y = 500e^{-.032t} \quad y = 500e^{-.032(20)} \approx 263.65 grams$

b) Approximate the half-life of lead-210. That means to determine the amount of time required for there to be half of the original amount of lead. Round to the nearest tenth.

$250 = 500e^{-.032t}$

$\frac{1}{2} = e^{-.032t}$

$\ln(1/2) = \ln(e^{-.032t})$

$\ln(1/2) = -.032t$

$t = \frac{\ln(1/2)}{-.032} \approx 21.66 yrs.$