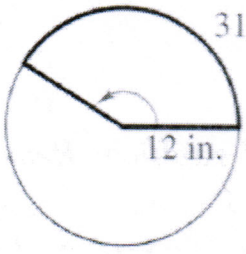
Precalculus with Limits
Mixed Practice

Name Solutions

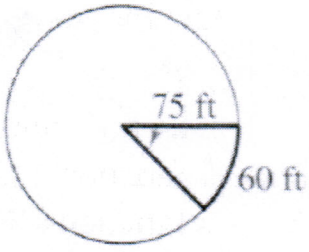
Period

1. Find the measure of the angle (in radians) that subtends each arc.



$S = r\theta$

$31 = 12 \cdot \theta \rightarrow \theta = \frac{31}{12}$



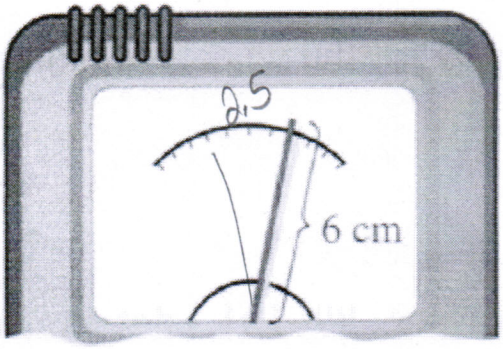
$S = r\theta$

$60 = 75\theta$

$\frac{60}{75} = \frac{4}{5} = \theta$

2.

Instrumentation A voltmeter's pointer is 6 centimeters in length (see figure). Find the angle through which it rotates when it moves 2.5 centimeters on the scale.



$S = r\theta$

$2.5 = 6 \cdot \theta$

$\frac{2.5}{6} = \theta = \frac{5}{12}$

3. A satellite in circular orbit 1250 kilometers above Earth makes one complete revolution every 110 minutes. What is its linear speed? Assume that the Earth is a sphere of radius 6400 kilometers.

$r = 6400 + 1250 = 7650$

$\frac{1\,rev}{110\,min} \cdot \frac{2\pi\,rad}{1\,rev} = \frac{2\pi}{110} = \frac{\pi}{55} = \omega$

$v = r\omega = 7650\left(\frac{\pi}{55}\right) \approx 437 \frac{km}{min}$.

4. The circular blade on a saw has a diameter of 7.5 inches and rotates at 2400 revolutions per minute.

a) Find the angular speed in radians per second.

$2400\frac{rev}{min} \cdot \frac{2\pi\,rad}{1\,rev} \cdot \frac{1\,min}{60\,sec} = \frac{4800\pi}{60} = 80\pi \frac{rad}{sec}$

b) Find the linear speed of the saw teeth (in feet per second) as they contact the wood being cut.

$r = 3.75\,in \cdot \frac{1\,ft.}{12\,in} = .3125\,ft.$

$v = .3125(80\pi) \approx 78.54\,ft./sec.$

5. A car is moving at a rate of 40 miles per hour, and the diameter of its wheels is 2.5 feet.

a) Find the number of revolutions per minute the wheels are rotating.

$v = 40\frac{miles}{hour} \cdot \frac{5280\,ft.}{1\,mile} \cdot \frac{1\,hour}{60\,min.} = 3520\frac{ft.}{min.}$ $\quad r = 1.25$

$v = r\omega \rightarrow 3520 = 1.25 \cdot \omega \rightarrow \omega = 2816\frac{rad.}{min.}$

$2816\frac{rad}{min} \cdot \frac{1\,rev}{2\pi\,rad} \approx 448\,rpm$

b) Find the angular speed of the wheels in radians per minute.

$2816\frac{rad}{min}$

6. Rewrite each angle in degree measure.

a) $\frac{3\pi}{2}$ 270°

b) $-\frac{7\pi}{6}$ -210°

c) -4π -720°

d) 3π 540°

e) $\frac{7\pi}{3}$ 420°

f) $-\frac{13\pi}{60} \cdot \frac{180^\circ}{\pi}$ -39°

g) $-\frac{15\pi}{6} \cdot \frac{180^\circ}{\pi}$ -450°

h) $\frac{28\pi}{15} \cdot \frac{180^\circ}{\pi}$ 336°

7. Rewrite each angle in radian measure. Leave your answer in terms of pi. No decimal answers.

a) $30^\circ \cdot \frac{\pi}{180}$ $\frac{\pi}{6}$

b) 150° $\frac{5\pi}{6}$

c) 315° $\frac{7\pi}{4}$

d) $-240^\circ \cdot \frac{\pi}{180}$ $-\frac{4\pi}{3}$

e) $-270^\circ \cdot \frac{\pi}{180}$ $-\frac{3\pi}{2}$

f) $-20^\circ \cdot \frac{\pi}{180}$ $-\frac{\pi}{9}$

8. Determine the quadrant in which each angle lies.

a) $-\frac{2\pi}{5} \cdot \frac{180^\circ}{\pi}$ -72° IV

b) $-\frac{15\pi}{4} \cdot \frac{180^\circ}{\pi}$ -675° I

c) $\frac{23\pi}{6} \cdot \frac{180^\circ}{\pi}$ 690° IV

Evaluate the following trig functions using the unit circle. Rationalize any denominators.

$\sin\left(-\frac{\pi}{3}\right) = \frac{-\sqrt{3}/2}{}$

$\csc\left(-\frac{\pi}{3}\right) = \frac{-2/\sqrt{3} = -2\sqrt{3}/3}{}$

$\cos\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}/2}{}$

$\sec\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{}$

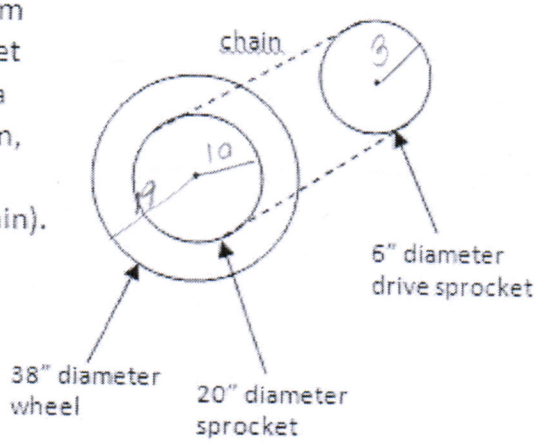
$\tan\left(-\frac{5\pi}{6}\right) = \frac{1/2}{\sqrt{3}/2} = \sqrt{3}/3$

$\cot\left(-\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{}$

9.

Old-fashioned trucks used a chain to transmit power from the engine to the wheels. Suppose that the drive sprocket had a diameter of 6 inches and the wheel sprocket had a diameter of 20 inches. If the drive sprocket goes 300 rpm, find:

a. The angular velocity of the drive sprocket (rad/min).
b. The linear velocity of the 20-inch wheel sprocket (inch/min).
c. The angular velocity of the wheel (rad/min).
d. The speed of the truck (mi/h).



1 mile = 63360 inches

a) $300 \frac{rev}{min} \cdot \frac{2\pi}{1 rev.} = 600\pi$

b) $3 \cdot 600\pi = 1800\pi \approx 5654.87 \frac{in}{min}$

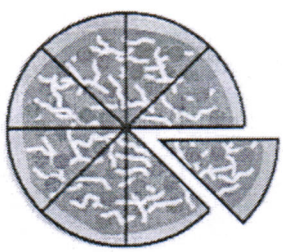
c) angular velocity $= \frac{v}{r} = \frac{5654.87}{10} = 565.49 \frac{rad}{min}$

d) $v = 19 \cdot 565.49 = 10{,}744.25 \frac{in}{min} \cdot \frac{1 mi}{63360 in} \cdot \frac{60 min}{1 hr.}$

$v \approx 10.17 mph$

10.

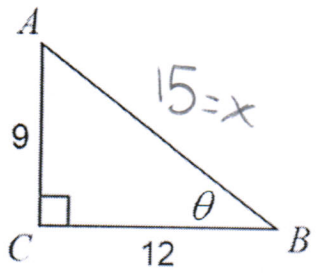
. **PIECES OF PIZZA** Suppose the pizza shown is divided into 8 equal pieces. The diameter of the pizza is 16 inches. What is the area of one piece of pizza?



$r = 8 in.$ $A = \pi \cdot 64$

$8\pi \approx 25.13 in^2$

11. Find the value of all six trig functions for angle θ.



$9^2+12^2=x^2$

$x^2=225$

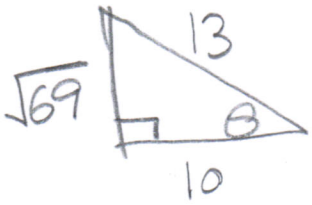
$x=15$

$\sin\theta=\frac{9}{15}$ $\quad \csc\theta=\frac{15}{9}$

$\cos\theta=\frac{12}{15}$ $\quad \sec\theta=\frac{15}{12}$

$\tan\theta=\frac{9}{12}$ $\quad \cot\theta=\frac{12}{9}$

12. Find the value of the remaining trig functions given that $sec\theta=\frac{13}{10}$. $=\frac{hyp}{adj.}$



$\sin\theta=\frac{\sqrt{69}}{13}$ $\quad \csc\theta=\frac{13\sqrt{69}}{69}$

$\cos\theta=\frac{10}{13}$ $\quad \sec\theta=\frac{13}{10}$

$\tan\theta=\frac{\sqrt{69}}{10}$ $\quad \cot\theta=\frac{10\sqrt{69}}{69}$

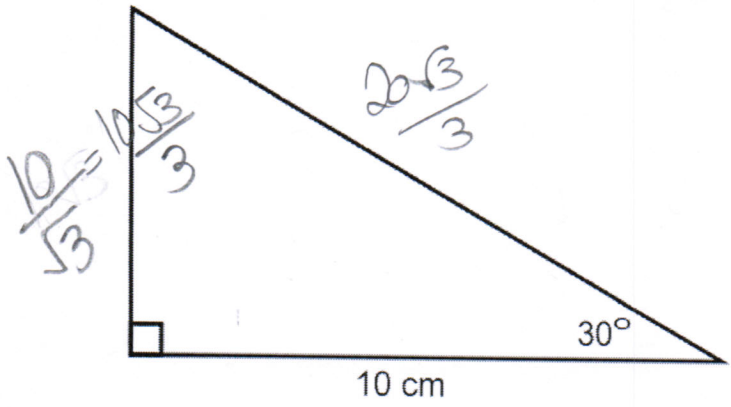
13. The radius of the Earth is about 3,959 miles (assuming that it is a perfect sphere). If a person is standing still at the equator, what is their linear speed as the Earth rotates on its axis? Why do you think we launch space vehicles from Florida and not somewhere like Ohio?

$\frac{1\ rev}{24\ hrs}\cdot\frac{2\pi\ rad}{1\ rev.}=\frac{2\pi}{24}=\frac{\pi}{12}\ \frac{rad.}{hr.}=\omega$

$v=r\omega=3{,}959\left(\frac{\pi}{12}\right)\approx 1036.4638\ mph$

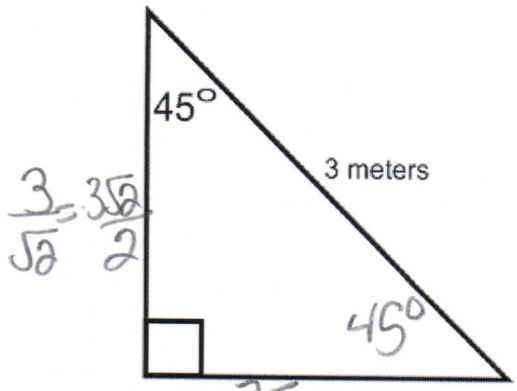
↑ wow!

14. Find the missing sides of each triangle.



$\frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$

$\frac{20\sqrt{3}}{3}$

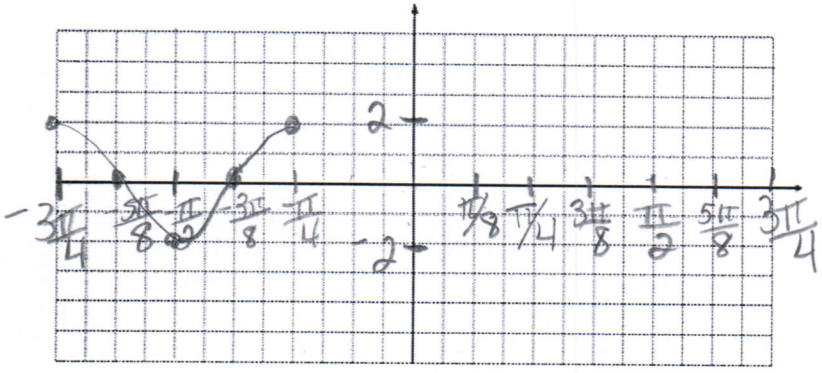


$\frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$\frac{3\sqrt{2}}{2}$

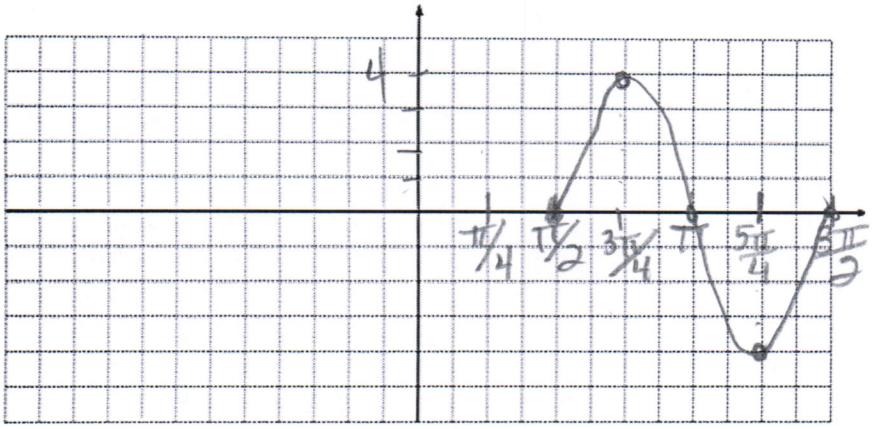
15. $y = 2\cos\left(4\left(x + \frac{3\pi}{4}\right)\right)$

$\text{period} = \frac{2\pi}{4} = \frac{\pi}{2}$



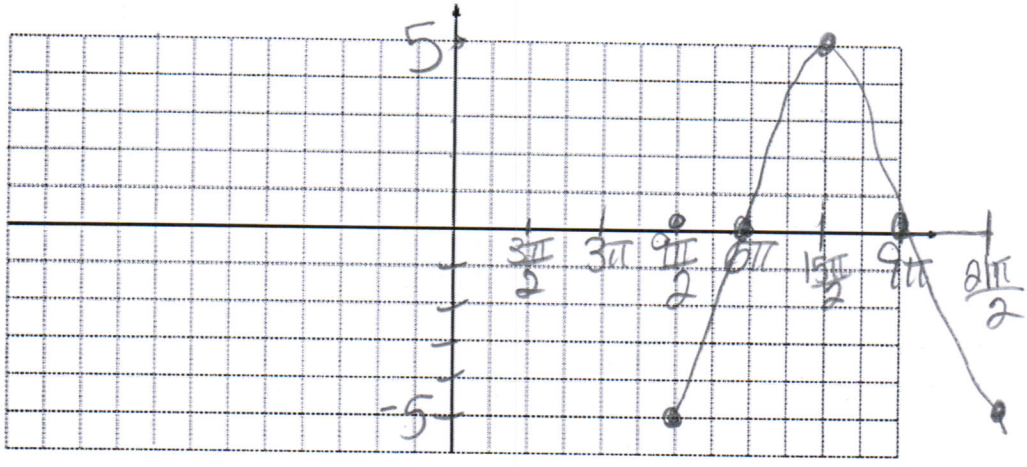
16. $y = 4\sin\left(2\left(x - \frac{\pi}{2}\right)\right)$

$\text{period} = \frac{2\pi}{2} = \pi$



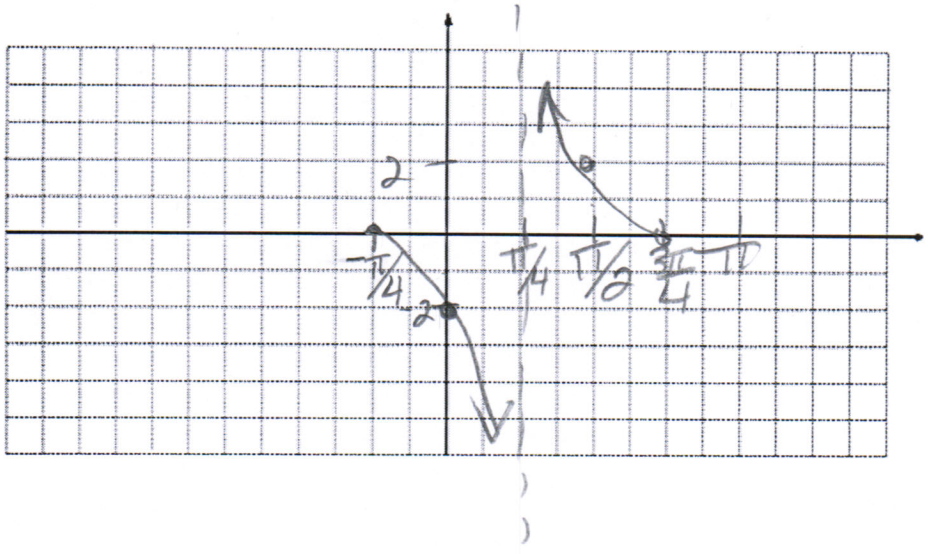
17. $y = -5\cos\left(\frac{1}{3}\left(x - \frac{9\pi}{2}\right)\right)$

Period $= \frac{2\pi}{1/3} = 6\pi$



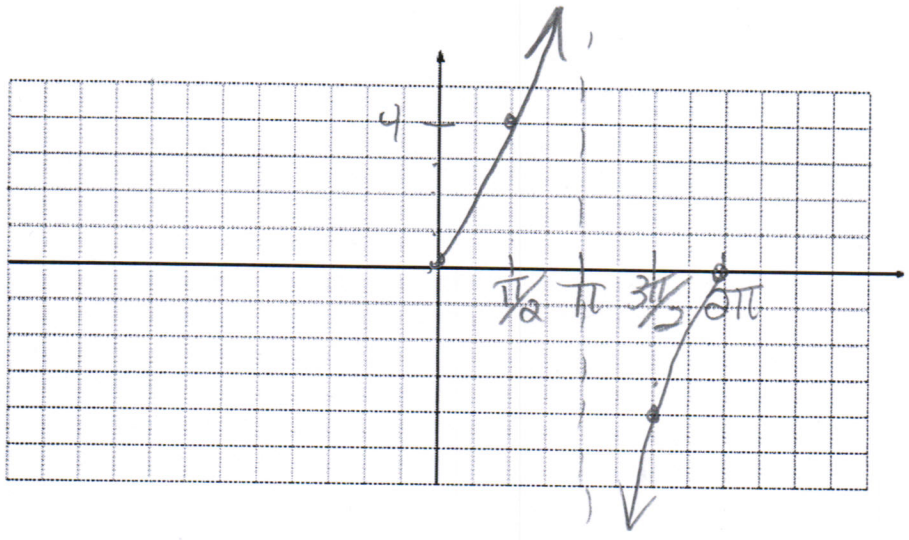
18. $y = -2\tan\left(x + \frac{\pi}{4}\right)$

Period $= \pi$



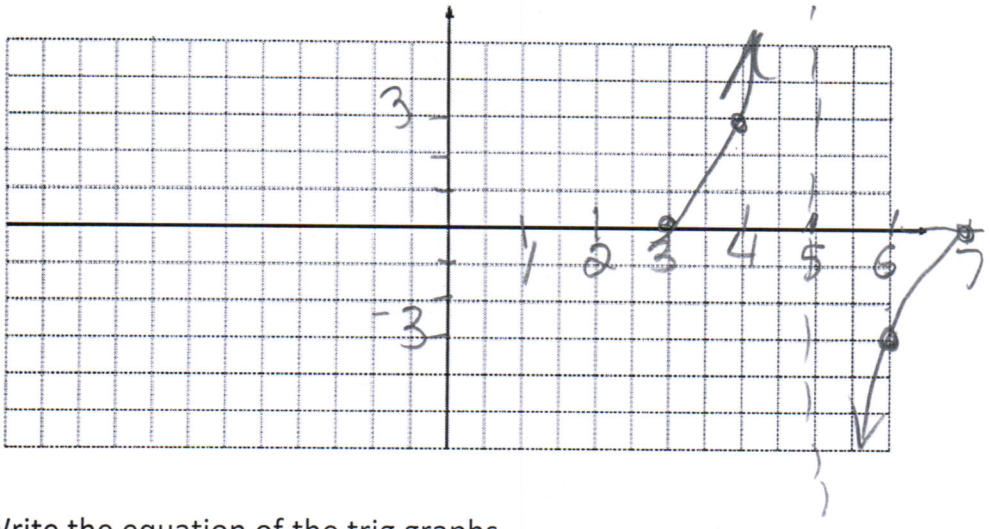
19. $y = 4\tan\left(\frac{1}{2}x\right)$

period $= \frac{\pi}{1/2} = 2\pi$



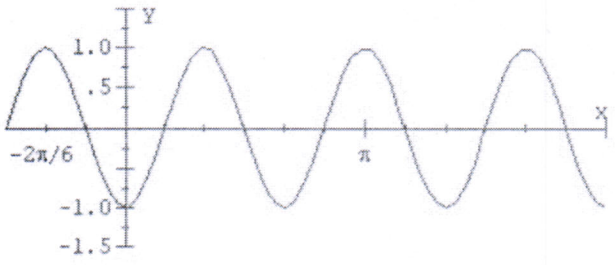
20. $y = 3\tan\left(\frac{\pi}{4}(x-3)\right)$

period $= \frac{\pi}{\pi/4} = \pi \cdot \frac{4}{\pi} = 4$



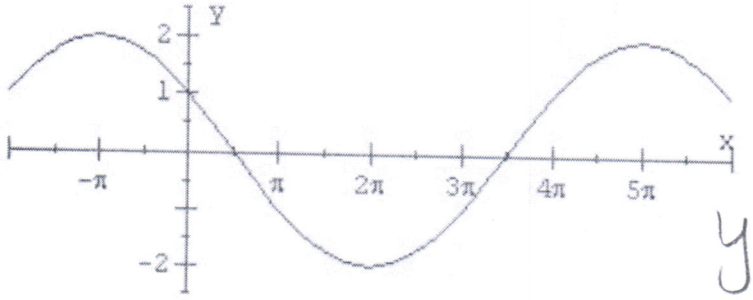
Write the equation of the trig graphs.

21.



$y = -\cos(3x)$

22.



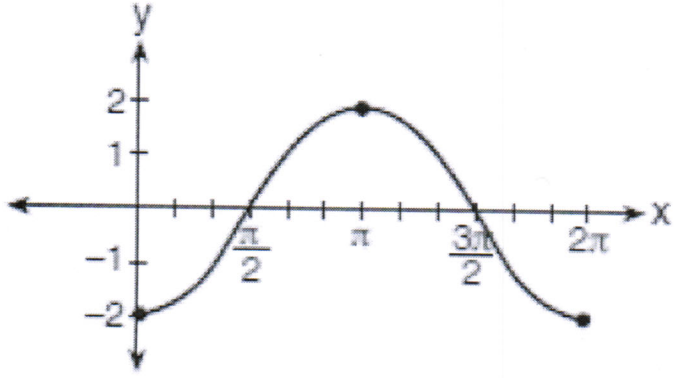
2 possibilities (there are more though)

$y = 2\cos(1/3(x+\pi))$

or

$y = -2\sin(1/3(x-\pi/2))$

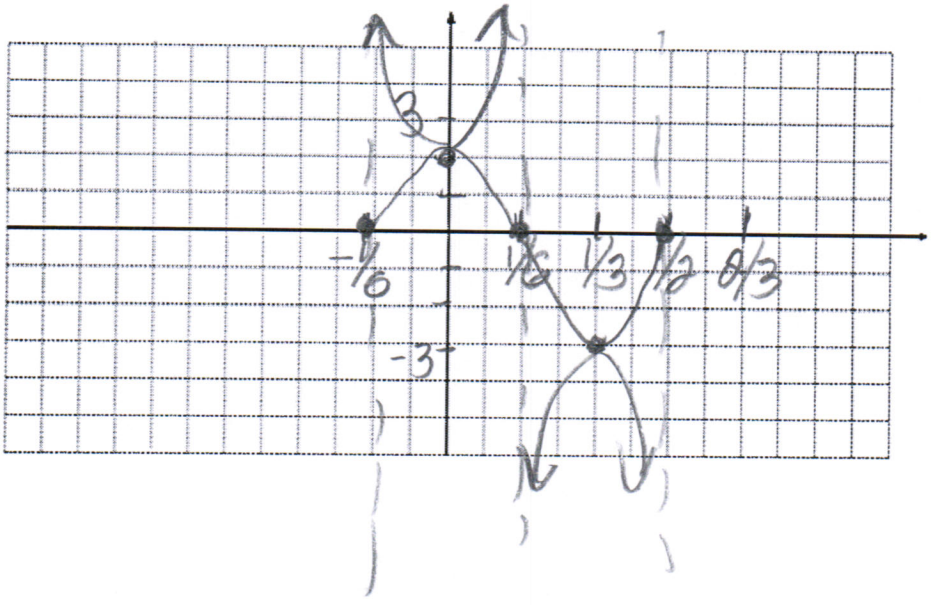
23.



$y = -2\cos(x)$

24. Graph $y = 3\csc\left(3\pi\left(x+\frac{1}{6}\right)\right)$

$y = 3\sin(3\pi(x+1/6))$



$\text{period} = \frac{2\pi}{3\pi} = \frac{2}{3}$